A Theory Appendix: General Framework

A.1 Notation and Definitions

Let X be a set.

Measures $\mathcal{B}(X)$ denotes the Borel-algebra over X and $\mathcal{P}(X)$ denotes the space of probability measures over X. A function $Q : X \times \mathcal{B}(X) \to [0,1]$ is a transition function if $Q(x;\cdot)$ is a probability measure for all $x \in X$ and $Q(\cdot;A)$ is measurable for all $A \in \mathcal{B}(X)$:

Sequences. A finite sequence $(x_1, x_2, \dots, x_t)$ such that $x_t \in X$ for all $t = 1, 2, \dots$ is denoted with x^t and is an element of $X^t = \times_{\tau=1}^{t} X$; an infinite sequence with $t \to \infty$ is denoted with $\bar{x}$ and is an element of X^∞.

Topology If X is discrete, it is endowed with the discrete topology. The set $\mathcal{P}(X)$ is endowed with the topology of weak convergence.

Partial orders All partial orders will be denoted by $\geq$. Suppose that $(X, \geq)$ is a partially-ordered set. For any two sequences $x^{1,t}, x^{2,t} \in X^t$, we say that $x^{2,t} \geq x^{1,t}$ if $x_{2,t} \geq x_{1,t}$ pointwise. The set $\mathcal{P}(X)$ is endowed with the stochastic dominance order, i.e., for $p_1, p_2 \in \mathcal{P}(X)$,

$$p_2 \geq p_1 \iff \int_X f(x) p_2(\mathrm{d}x) \geq \int_X f(x) p_1(\mathrm{d}x)$$

for all increasing and integrable functions $f : X \to \mathbb{R}$.

Monotonicity In the following, we will often deal with sequences of functions $f_t : X^t \to Y$ for $t = 1, 2 \dots$ In this case we will say that the sequence $\bar{f}$ is increasing in $\bar{x} \in X^\infty$ if each f_t is increasing with respect to $x^t \in X^t$, where x^t is the partial history up to time t in $\bar{x}$.

A.2 Setup

A social group consists of a continuum of individuals. Let $y \in \{0, 1\}$ be an action, $x \in X \subset \mathbb{R}$ be an observable individual type, and $\varepsilon \in \mathcal{E}$ be a vector of choice-specific unobserved idiosyncratic shocks. Let $p \in \mathcal{P}$ be a probability measure over X, and let λ be the (invariant) probability measure over $\mathcal{E}$. Let $\theta \in \Theta \subset \mathbb{R}$ be the value of an aggregate stochastic shock that is common across individuals, and, finally, let $z \in Z \subset \mathbb{R}$ be the value of an exogenous parameter.

The **flow payoff** for an individual is a function:

$$\tilde{u} : \{0,1\} \times X \times \mathcal{E} \times \mathcal{P} \times \Theta \times Z \to \mathbb{R}$$

Time is discrete and runs from $t = 0$ to infinity. A **partial history** at time t is a sequence $\{\theta_\tau)\}_{\tau=1}^{t} \in \Theta^t$, with $\Theta^t = \times_{\tau=1}^{t}\Theta$ denoting the set of all possible partial histories up to time t.

If an individual of type $x \in X$ chooses action $y \in \{0,1\}$, her type in the next period is sampled from a distribution $Q(x,d;\cdot)$, where Q is a transition kernel $Q : X \times \{0,1\} \times \mathcal{B}(X) \to [0,1]$. If the partial history at time t is $\theta^t \in \Theta^t$, the aggregate shock in the next period is sampled from a probability measure $q_t(\theta^t)$, where q_t is a map from Θ^t into $\mathcal{P}(\Theta)$. By contrast, agents have perfect foresight about the sequence $\bar{z} = \{z_t\}_{t=1}^{\infty}$.

A **strategy** at time t is a function $w_t : X \times \mathcal{E} \times \Theta^t \to \{0,1\}$ such that w_t is measurable on the relevant σ-algebra. Given the properties of Q and λ, only the current value of the individual state variables $x \in \mathcal{X}$ and $\varepsilon \in \mathcal{E}$ matter for the individual's decisions. A strategy **plan** is a sequence of strategies $\bar{w} = \{w_1, w_2, \ldots\}$. Let W^∞ be the set of all possible plans, and let $0 < \beta < 1$ be the **discount factor**. The agent's problem is to select a plan to maximize the expected discounted sum of her future payoffs

$$\max_{\bar{w} \in W^\infty} \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \tilde{u}(y_t, x_t, \varepsilon_t, p_t, \theta_t, z_t) \tag{12}$$

with $p_0 \in \mathcal{P}$ and $\theta_0 \in \Theta$ given.

A.3 Conditional No Aggregate Uncertainty

A plan $\bar{w} \in W^\infty$ and the transition kernel Q induce a law of motion for the probability measure over X. Because $\bar{w}$ is a sequence of random variables, the sequence of measures over X is also stochastic. Following Jovanovic and Rosenthal (1988), Bergin and Bernhardt (1992), Miao (2006) and others, we assume that this stochastic process satisfies the **no aggregate uncertainty** condition.

Specifically, let $\theta^t \in \Theta^t$ be the observed partial history at time t. For a given strategy w_t, let $\hat{Q}_t^w : X \times \Theta^t \times \mathcal{B}(X) \to [0,1]$ be defined from:

$$\hat{Q}_t^w(x, \theta^t; A) = \lambda\Big(\{\varepsilon \in \mathcal{E} : w_t(x, \varepsilon, \Theta^t) = 1\}\Big)Q(x,1;A) + \lambda\Big(\{\varepsilon \in \mathcal{E} : w_t(x, \varepsilon, \Theta^t) = 0\}\Big)Q(x,0;A) \tag{13}$$

for all $x \in X$ and $A \in \mathcal{B}(X)$. With no aggregate uncertainty, we have:

$$p_{t+1}(\cdot) = \int_X \hat{Q}_t^w(x, \theta^t; \cdot) p_t(\mathrm{d}x), \quad t = 1, 2 \ldots$$

Because this holds for each $\theta^t \in \Theta^t$, we can construct a map $\pi_t : \Theta^t \to \mathcal{P}$ for each time $t > 0$. Let $\mathcal{F}_t$ be the space of such functions and let $\bar{\pi} \in \mathcal{F}^\infty$, with $\mathcal{F}^\infty = \times_{t=1}^\infty \mathcal{F}_t$. While the sequence of measures $\{p_1, p_2 \ldots\}$ is stochastic, the sequence $\bar{\pi} = \{\pi_1, \pi_2 \ldots\}$ is not and, as a result, it can be included as a parameter in the agent's problem.

A.4 The Leader

In the following, we will assume that the stochastic process for the aggregate shocks is the outcome of the optimizing behavior of another agent, called the leader. In particular, we assume that the leader also solves Problem 12, with $\theta \in \Theta$ being her individual type. Therefore a strategy for the leader at time t is a measurable function $w_t^\ell : \Theta^t \times \mathcal{E} \to \{0, 1\}$ and a leader's plan $\bar{w}^\ell$ is a sequence $\{w_t^\ell\}_{t=1}^\infty$. Let $W^{\ell,\infty}$ denote the space of all possible leader's plans. We shall impose the following restriction on Θ:

Assumption A.1. *The set Θ is countable.*

Let $Q^\ell : \Theta \times \{0, 1\} \times \mathcal{B}(\Theta) \to [0, 1]$ be the transition kernel defined on Θ for the leader. A leader's plan $\bar{w}^\ell$ and the transition kernel Q^ℓ induce a probability measure over the value of the leader's type in the next period. Let $\hat{Q}_t^{w^\ell} = \Theta^t \times \mathcal{B}(\Theta^t) \to [0, 1]$ be defined from:

$$\hat{Q}_t^{w^\ell}(\theta^t; A) = \lambda\Big(\{\varepsilon \in \mathcal{E} : w_t^\ell(\theta^t, \varepsilon) = 1\}\Big) Q^\ell(\theta_t, 1; A) + \lambda\Big(\{\varepsilon \in \mathcal{E} : w_t^\ell(\theta^t, \varepsilon) = 0\}\Big) Q^\ell(\theta_t, 0; A) \tag{14}$$

Ordinary agents take $\bar{w}^\ell$ as given when they solve Problem 12. Under rational expectations,

$$q_t(\theta^t)(A) = \hat{Q}_t^{w^\ell}(\theta^t; A) \quad \text{all } t = 1, 2 \ldots \tag{15}$$

for all $A \in \mathcal{B}(\Theta^t)$ and all $t => 0$.

A.5 Dynamic Programming

We formulate problem (12) as a non-stationary dynamic programming problem. Since $\bar{\pi}$, $\bar{w}^\ell$, and $\bar{z}$ are held fixed in this subsection, we temporarily drop them from the notation.

Let $V_t : X \times \mathcal{E} \times \Theta^t \to \mathbb{R}$ denote the value function at time t for all $t = 1, 2 \ldots,$ and let the sequence $\bar{V} = \{V_t\}_{t=1}^{\infty}$ be defined recursively from

$$\begin{aligned} V_t(x, \varepsilon, \theta^t) = \max_{y \in \{0,1\}} &\ u(y, x, \theta_t, \pi_t(\theta^{t-1}), z_t, \varepsilon) \\ &+ \beta \int_X \int_\Theta \int_{\mathcal{E}} V_{t+1}(x', \varepsilon', \theta^{t+1}) \lambda(\mathrm{d}\varepsilon') Q(x, y; \mathrm{d}x')[q_t(\theta^t)](\mathrm{d}\theta_{t+1}) \end{aligned} \tag{16}$$

Following the standard approach in dynamic discrete choice modeling (Aguirregabiria and Mira, 2010), it is convenient to work with the ex ante value function, i.e., the value of the problem before observing the idiosyncratic shocks:

$$\begin{aligned} \hat{V}_t(x, \theta^t) = \int_{\mathcal{E}} \Bigg\{ &\max_{y \in \{0,1\}} u(y, x, \theta_t, \pi_t(\theta^{t-1}), z_t, \varepsilon) \\ &+ \beta \int_X \int_\Theta \hat{V}_{t+1}(x', \theta^{t+1}) Q(x, y; \mathrm{d}x')[q_t(\theta^t)](\mathrm{d}\theta_{t+1}) \Bigg\} \lambda(\mathrm{d}\varepsilon) \end{aligned} \tag{17}$$

Let $\hat{\mathcal{V}}^\infty$ denote the space of sequences of bounded continuous functions $\hat{V}_1, \hat{V}_2, \ldots,$ such that $\hat{V}_t : X \times \Theta^t \to \mathbb{R}$ for for all $t = 1, 2, \ldots,$ and define the sup norm on $\hat{\mathcal{V}}^\infty$ from

$$\|\hat{V}^\infty\| = \sup_{t, x, \theta^t} \hat{V}_t(x, \theta^t)$$

. We say that the payoff function $\tilde{u}$ is additive in the idiosyncratic shock if $u : \{0,1\} \times X \times \mathcal{P} \times \Theta \times Z \to \mathbb{R}$ exists such that $\tilde{u}(y, x, \theta, \varepsilon) = u(y, x, \theta) + \varepsilon(y)$ on $\{0,1\} \times X \times \Theta \times \mathcal{E}$, where $\varepsilon(y) \in \mathbb{R}$ denotes the element of vector ε element associated with choice $y \in \{0,1\}$.

Assumption A.2. *The set X is a compact. The payoff function $\tilde{u}$ is jointly continuous and additive in the idiosyncratic shock. The discount factor $\beta \in [0,1]$. The transition kernel Q has the Feller property. The density λ is continuous and its first moment is bounded.*

The following Proposition adapts the arguments in Rust (1988) to the non-stationary setting studied in Jovanovic and Rosenthal (1988).

Proposition A.1. *Under Assumption A.2, there exists a unique sequence $\hat{V}_\infty$ satisfying (16).*

Proof. Write the system defined by (17) in vector form as $\hat{V}^\infty = T\hat{V}^\infty$, such that the operator T is defined from the right-hand side of (17). The proof verifies that the operator T satisfies the conditions of the contraction mapping theorem under Assumption A.2.

First, we show $T : \hat{\mathcal{V}}_\infty \to \hat{\mathcal{V}}_\infty$. Take $\hat{V}^\infty \in \hat{\mathcal{V}}^\infty$ and consider a generic period $t > 0$. Because Q has the Feller property, the map

$$x \mapsto \int_X \hat{V}_{t+1}(x', \theta^{t+1}) Q(x, y; \mathrm{d}x')$$

is continuous in $x \in X$ for each $\theta^{t+1} \in \Theta^{t+1}$ and $y \in \{0, 1\}$. Therefore the map

$$(x, y, \theta^t) \mapsto \int_\Theta \int_X \hat{V}_{t+1}(x', \theta^{t+1}) Q(x, y; \mathrm{d}x')[q(\theta^t)](\mathrm{d}\theta_{t+1})$$

is continuous with the discrete topology on Θ^t and $\{0, 1\}$ and bounded because it is a continuous function on a compact set. The payoff function u is also bounded for the same reason. The choice set $\{0, 1\}$ is trivially continuous and compact-valued. Therefore the maximand in equation (17) satisfies the conditions of Berge's maximum theorem, and the map:

$$\begin{aligned}(x, \theta^t) \mapsto \max_{y \in \{0,1\}} & \, u(y, x, \theta_t) + \varepsilon(y) \\ & + \beta \int_X \int_\Theta \hat{V}_{t+1}(x', \theta^{t+1}) Q(x, d; \mathrm{d}x')[q_t(\theta^t)](\mathrm{d}\theta_{t+1})\end{aligned}$$

is continuous. Because λ is continuous with a finite first moment, $\hat{V}_t$ in 17 is continuous and bounded. Because this argument holds for all $t = 1, 2 \ldots$, the operator T maps $\hat{V}^\infty$ into itself.

Second, we show that T is a contraction operator. Consider two generic sequences $\hat{V}^\infty, \hat{W}^\infty \in \hat{\mathcal{V}}^\infty$. Then we have:

$$\begin{aligned}(T\hat{V}^\infty)_{t+1}(x, \theta^t) & - (T\hat{W}^\infty)_{t+1}(x, \theta^t) \\ & \leq \max_{y \in \{0,1\}} \beta \left| \mathbb{E}_{Q,q}\left[\hat{V}_{t+1}(x', \theta^{t+1})\right] - \mathbb{E}_{Q,q}\left[\hat{W}_{t+1}(x', \theta^{t+1})\right]\right| \\ & \leq \beta \|\hat{V}^\infty - \hat{W}^\infty\|\end{aligned}$$

for all $t = 1, 2, \ldots$, $x \in X$, and $\theta^t \in \Theta^t$. Therefore

$$\|T\hat{V}^\infty - T\hat{W}^\infty\| \leq \beta \|\hat{V}^\infty - \hat{W}^\infty\|$$

and T is a contraction operator.

Third, $\hat{V}_\infty$ is a complete metric space with the sup norm defined above. Then the conclusion follows by applying the contraction mapping theorem. $\square$

A.6 The Single Agent Problem

At each time t, let $G_t : X \times \mathcal{E} \times \Theta^t \times \mathcal{F}^\infty \times Z^\infty \to \{0,1\}$ denote the optimal non-stationary policy correspondence associated with the recursive problem (16):

$$G_t(x,\varepsilon,\theta^t,\bar{\pi},\bar{w}^\ell,\bar{z}) = \arg\max_{y\in\{0,1\}} u(y,x,\theta^t,\pi_t(\theta^{t-1}),z_t) + \varepsilon(y) \\ +\beta \int_\Theta \int_X \hat{V}_{t+1}(x',\theta^{t+1},\bar{\pi},\bar{w}^\ell,\bar{z}) Q(x,y;\mathrm{d}x')[q_t(\theta^t)](\mathrm{d}\theta_{t+1}) \tag{18}$$

Lemma A.1. *Under Assumption A.2, the set:* $\{\varepsilon \in \mathcal{E} : G_t(x,\varepsilon,\theta^t,\bar{\pi},\bar{w}^\ell,\bar{z}) = \{0,1\}\}$ *has measure zero for all* $x \in X, \theta^t \in \Theta^t$, $\bar{\pi} \in \mathcal{F}^\infty$, $\bar{w}^\ell \in W^\ell$, $\bar{z} \in Z^\infty$, *and* $t > 0$.

Proof. The additive payoff structure implies

$$G_t(x,\varepsilon^*,\theta^t,\bar{\pi},\bar{w}^\ell,\bar{z}) = \{0,1\} \iff \\ \varepsilon^* \equiv \varepsilon(0) - \varepsilon(1) = \\ u(1,x,\theta^t,\pi_t(\theta^{t-1}),z_t) + \beta \int_\Theta \int_X \hat{V}_{t+1}(x',\theta^{t+1},\bar{\pi},\bar{w}^\ell,\bar{z}) Q(x,1;\mathrm{d}x')[q_t(\theta^t)](\mathrm{d}\theta_{t+1}) \\ -u(0,x,\theta^t,\pi_t(\theta^{t-1}),z_t) + \beta \int_\Theta \int_X \hat{V}_{t+1}(x',\theta^{t+1},\bar{\pi},\bar{w}^\ell,\bar{z}) Q(x,0;\mathrm{d}x')[q_t(\theta^t)](\mathrm{d}\theta_{t+1})$$

Because the density λ is continuous, the set $\{\varepsilon^*\}$ has measure zero. □

A policy function $g_t : X \times \mathcal{E} \times \Theta^t \times \mathcal{F}^\infty \times Z^\infty \to \{0,1\}$ at time t is a selection from G_t. For each selection g_t of G_t, define $\hat{g}_t : X \times \times \Theta^t \times \mathcal{F}^\infty \times Z^\infty \to \{0,1\}$ from:

$$\hat{g}_t\left(x,\theta^t,\bar{\pi},\bar{w}^\ell,\bar{z}\right) = \lambda\Big(\{\varepsilon \in \mathcal{E} : g_t(x,\varepsilon,\theta^t,\bar{\pi},\bar{w}^\ell,\bar{z}) = 1\}\Big) \quad \text{all } t = 1,2\ldots \tag{19}$$

Lemma A.1 implies that $\hat{g}_t$ is uniquely defined because the policy functions in G_t only differ on a set with measure zero with respect to λ. For the same reason, $\hat{g}_t$ is continuous.

Corollary A.1. *Under Assumptions A.2 ,* $\hat{g}_t$ *in* (19) *is jointly continuous in* $x \in X$ *and in* $\bar{\pi} \in \mathcal{F}^\infty$ *for all* $t = 1, 2, \ldots$

Proof. Because $\bar{w}^\ell$ and $\bar{z}$ are held constant throughout the proof, we temporarily drop them from the notation. Fix $t > 0$ and $\theta^t \in \Theta^t$, and take a sequence $\{x^n, \bar{\pi}^n\}$ in $X \times \mathcal{F}^\infty$ with $(x^n,\bar{\pi}^n) \to (x,\bar{\pi})$. Let the indicator functions $h_n : \mathcal{E} \to \{0,1\}$ and $h : \mathcal{E} \to \{0,1\}$, respectively, be defined from

$$h^n(\varepsilon) = \chi_{\{\varepsilon' \in \mathcal{E}: g_t(x^n,\theta^t,\bar{\pi}^n,\varepsilon')=1\}}(\varepsilon) \quad \text{and} \quad h(\varepsilon) = \chi_{\{\varepsilon' \in \mathcal{E}: g_t(x,\theta^t,\bar{\pi},\varepsilon')=1\}}(\varepsilon)$$

Clearly, $\hat{g}_t(x^n, \theta^t, \bar{\pi}^n) = \int_{\mathcal{E}} h^n(\varepsilon)\lambda(\mathrm{d}\varepsilon)$ and $\hat{g}_t(x, \theta^t, \bar{\pi}) = \int_{\mathcal{E}} h(\varepsilon)\lambda(\mathrm{d}\varepsilon)$. Then h^n is a sequence of bounded integrable functions. Because g_t is continuous in $(x, \bar{\pi})$ except on a measure zero set (Lemma A.1), h^n converges to h almost everywhere with respect to λ. By the Dominated Convergence Theorem,

$$\int_X h^n(\varepsilon)\lambda(\mathrm{d}\varepsilon) \to \int_X h^n(\varepsilon)\lambda(\mathrm{d}\varepsilon)$$

Because the choice of t and θ^t was arbitrary, the result holds for all $t > 0$ and $\theta^t \in \Theta^t$. □

Next, we introduce two assumptions concerning the supermodularity properties of the payoff function u and the transition kernel Q.

Assumption A.3. *The payoff function has increasing differences pairwise in (y, x, p) on $\{0, 1\} \times X \times \mathcal{P}$ and weakly increasing in $x \in X$.*

Assumption A.4. *The transition kernel Q is stochastically increasing in $y \in \{0, 1\}$ and $x \in X$, and exhibits stochastically increasing differences in (y, x) on $\{0, 1\} \times X$.*

Under these assumptions, we can prove the following proposition.

Proposition A.2. *Under Assumptions A.2-A.4 the optimal policy correspondence in (18) is increasing in $x \in X$ and in $\bar{\pi} \in \mathcal{F}^\infty$ for all $t = 1, 2, \ldots$.*

Proof of Proposition A.2. We adapt the arguments in the proof of Theorem B.2 in Acemoglu and Jensen (2015). Since $\bar{w}^\ell$ and $\bar{z}$ are held fixed, we temporarily drop them from the notation.

The proof of Proposition A.1 shows that the sequence $\hat{V}^\infty$ can be found as the limit of a contraction operator. Hence $\hat{V}^\infty$ can be found as the limit of the sequence $\{\hat{V}^{\infty,n}\}_{n=1}^\infty$ in $\hat{\mathcal{V}}^\infty$, such that each $\hat{V}^{\infty,n}$ is defined recursively from:

$$\begin{aligned} \hat{V}_t^{n+1}(x, \theta^t, \bar{\pi}) = \int_{\mathcal{E}} \Bigg\{ & \max_{y \in \{0,1\}} u(y, x, \theta_t, \pi_t(\theta^{t-1})) + \varepsilon(y) \\ & + \beta \int_\Theta \int_X \hat{V}_{t+1}^n(x', \theta^{t+1}, \bar{\pi}) Q(x, y; \mathrm{d}x')[q_t(\theta^t)](\mathrm{d}\theta_{t+1}) \Bigg\} \lambda(\mathrm{d}\varepsilon) \end{aligned} \tag{20}$$

for all $t = 1, 2, \ldots$, where each element of the sequence converges pointwise.

We prove the result with respect to $\bar{\pi}$. Start from an arbitrary $\hat{V}^{\infty,n}$ such that each $\hat{V}_t^n$ exhibits increasing differences in $(x, \bar{\pi})$. Consider $\hat{V}_{t+1}^n$ in (20). Because increasing differences are preserved by integration, the map

$$(x, \theta^t, \bar{\pi}) \mapsto \int_\Theta \hat{V}^n_{t+1}(x, \theta^{t+1}, \bar{\pi})[q_t(\theta^t)](\mathrm{d}\theta_{t+1})$$

exhibits increasing differences in $(x, \bar{\pi})$ at each $\theta^t \in \Theta^t$. Because Q is stochastically increasing in $y \in \{0, 1\}$, Lemma C.1 implies that the map

$$(y, x, \theta^t, \bar{p}) \mapsto \int_X \left\{ \int_\Theta \hat{V}^n_{t+1}(x', \theta^{t+1}, \bar{\pi})[q_t(\theta^t)](\mathrm{d}\theta_{t+1}) \right\} Q(x, y; \mathrm{d}y') \tag{21}$$

exhibits increasing differences in $(y, \bar{\pi})$ at each $x \in X$ and $\bar{\pi} \in \mathcal{F}^\infty$.

Because the payoff function exhibits increasing differences in $(y, \bar{\pi})$, and because the sum of supermodular functions is supermodular (Lemma 2.6.1, part (b) in Topkis (1998)), we conclude that the maximand in (20) exhibits increasing differences in $(y, \bar{\pi})$.

Next we show that $\hat{V}^{n+1}_t$ exhibits increasing differences in $(x, \bar{\pi})$. Because Q is stochastically increasing in $x \in X$, Lemma C.1 implies that the map in (21) exhibits increasing differences in $(x, \bar{\pi})$. Because the payoff function exhibits increasing differences in $(x, \bar{\pi})$, and because the sum of supermodular functions is supermodular, the maximand in (20) exhibits increasing differences in $(x, \bar{\pi})$. Because increasing differences are preserved under the max operator (Hopenhayn and Prescott, 1992, Lemma 1) and under the integral operator, we conclude that $\hat{V}^{n+1}_t$ exhibits increasing differences in $(x, \bar{\pi})$.

Because this holds for an arbitrary time index t, we have shown that all elements in $\hat{V}^{\infty, n+1}$ exhibit increasing differences in $(x, \bar{\pi})$. Applying this argument recursively, we find that all elements of the sequence $\hat{V}^{\infty,1}, \hat{V}^{\infty,2}, \ldots$ exhibit increasing differences in $(x, \bar{\pi})$ and so does the pointwise limit $\hat{V}^\infty = \lim_{n\to\infty} \hat{V}^{\infty,n}$ (Lemma 2.6.1, part (c) in Topkis (1998)).

Now define G_t as in (18). We have shown above that the maximand has increasing differences in $(y, \bar{\pi})$ for all $t > 0$. Hence by theorem 2.8.4 in Topkis (1998), G_t is increasing in $\bar{\pi} \in \mathcal{F}^\infty$.

To prove the result with respect to x, start from a sequence $\hat{V}^{\infty,n}$ such that $\hat{V}^n_t$ is increasing in $x \in X$ for all $t > 0$. Integrating over $\theta_{t+1} \in \Theta$ preserves the monotonicity. Then, because Q has stochastically increasing differences in (x, y) on $X \times \{0, 1\}$, it follows that (21) has increasing differences in (x, y) . Because the payoff function u has increasing differences in (y, x), we conclude that the maximand in (21) has increasing differences in (y, x).

To apply the argument recursively, we need to verify that $\hat{V}^{n+1}_t$ is increasing in x on X. This is the case because Q is stochastically increasing in $x \in X$, because the payoff function is (weakly) increasing in $x \in X$, and because the monotonicity is preserved under the max and the integral operators.

Because this holds for an arbitrary time index t, all elements in $\hat{V}^{\infty,n+1}$ are increasing in $x \in X$. Applying this argument recursively, we find that all elements of the sequence $\hat{V}^{\infty,1}, \hat{V}^{\infty,2}, \dots$ are increasing in $x \in X$ and so is the pointwise limit $\hat{V}^{\infty} = \lim_{n\to\infty} \hat{V}^{\infty,n}$ (Lemma 2.6.1, part (c) in Topkis (1998)).

Now define G_t as in (18). We have shown above that the maximand has increasing differences in (y, x) for all $t = 1, 2, \dots$ Hence by Theorem 2.8.4 in Topkis (1998), G_t is increasing in $x \in X$. □

The properties of the optimal correspondence carry on in a natural way to $\hat{g}_t$.

Corollary A.2. *Suppose that Assumption A.4 holds. If the optimal policy correspondence in* (18) *is increasing in* x*, then* $\hat{g}_t$ *in* (19) *is also increasing in* x *for all* $t > 0$*. If the optimal policy correspondence in* (18) *is increasing in* $\bar{\pi}$*, then* $\hat{g}_t$ *in* (19) *is also increasing in* $\bar{\pi}$ *for all* $t > 0$*.*

A.7 Aggregation for the Social Group

The policy correspondence in (18) describes the optimal behavior of an individual taking $\bar{\pi}$ as fixed. At the same time, as discussed above, a plan $\bar{w} \in W$ and the transition kernel Q induce a law of motion for the aggregate distribution over X. In this subsection, we ask: does $\bar{\pi}$ exist such that the solution to the individual problem given $\bar{\pi}$ also generates $\bar{\pi}$ ex post?

First, let us redefine (13) in terms of sequences of policy functions, rather than plans. Using (19), let $\hat{Q}_t : X \times \Theta^t \times \mathcal{F}^{\infty} \times W^{\ell,\infty} \times Z^{\infty} \times \mathcal{B}(X) \to [0, 1]$ be defined from:

$$\hat{Q}_t(x, \theta^t, \bar{\pi}, \bar{w}^{\ell}, \bar{z}; A) = \hat{g}_t(x, \theta^t, \bar{\pi}, \bar{w}^{\ell}, \bar{z})Q(x, 1; A) + \Big(1 - \hat{g}_t(x, \theta^t, \bar{\pi}, \bar{w}^{\ell}, \bar{z})\Big)\, Q(x, 0; A) \tag{22}$$

for all $x \in X$ and $A \in \mathcal{B}(X)$. The continuity properties of $\hat{Q}_t$ are a direct consequence of Corollary A.1 and the Assumption that Q has the Feller property.

Corollary A.3. *Under Assumption A.2, the transition kernel* $\hat{Q}_t : X \times \Theta^t \times \mathcal{F}^{\infty} \times W^{\ell,\infty} \times Z^{\infty} \times \mathcal{B}(X) \to [0, 1]$ *in* (22) *has the Feller property.*

Second, we define the map $B : \mathcal{F}^\infty \times W^{\ell,\infty} \times Z^\infty \to \mathcal{F}^\infty$ from:

$$B(\bar{\pi}, \bar{w}^\ell, \bar{z}) = \Big\{ \bar{\pi}' \in \mathcal{F}^\infty : [\pi'_{t+1}(\theta^t)](A) = \int_X \hat{Q}_t\big(x, \theta^t, \bar{\pi}, \bar{w}^\ell, \bar{z}; A\big)[\pi_t(\theta^{t-1})](\mathrm{d}x), \quad \text{all } A \in \mathcal{B}(X), \text{ all } \bar{\theta} \in \Theta^\infty, t = 1, 2, \ldots, p_0 \text{ given} \Big\}. \tag{23}$$

The map B takes a sequence $\bar{\pi} \in F^\infty$ and maps it into another sequence, $\bar{\pi}'$ generated as an outcome of the individual optimal choices given $\bar{\pi}$. The next two lemmas characterize the properties of B with respect to its argument $\bar{\pi}$.

Lemma A.2. *Under Assumptions A.2, B is continuous in $\bar{\pi}$.*

Proof. Consider a sequence $\{\bar{\pi}^n\}$ in $\mathcal{F}^\infty$ such that $\bar{\pi}^n \to \bar{\pi}$. Fix θ^t and t; we shall show

$$\lim_{n\to\infty} \int_X f(x') \int_X \hat{Q}^n_t\big(x, \bar{\pi}^n; \mathrm{d}x'\big)[\pi^n_t(\theta^{t-1})](\mathrm{d}x) = \int_X f(x') \int_X \hat{Q}_t\big(x, \bar{\pi}; \mathrm{d}x'\big)[\pi_t(\theta^{t-1})](\mathrm{d}x)$$

for every bounded and continuous function $f : X \to \mathbb{R}$. After exchanging the order of integration, we can express the above condition as

$$\lim_{n\to\infty} \int_X F_n(x)[\pi^n_t(\theta^{t-1})](\mathrm{d}x) = \int_X F(x)[\pi_{t+1}(\theta^t)](\mathrm{d}x)$$

$$\text{with} \quad F_n(x) = \int_X f(x')\hat{Q}_t\big(x, \bar{\pi}^n; \mathrm{d}x'\big), \quad F(x) = \int_X f(x')\hat{Q}_t\big(x, \bar{\pi}; \mathrm{d}x'\big)$$

Because $\hat{Q}_t$ has the Feller property, $F_n(x_n) \to F(x)$ whenever $x_n \to x$. Therefore the limit above holds by an application of Lemma C.5. Because the choice of θ^t and t was arbitrary, the result holds for all $\theta^t \in \Theta^t$ and all $t > 0$. □

Lemma A.3. *Under Assumptions A.2-A.4, B is increasing in $\bar{\pi}$.*

Proof. Because $\bar{w}^\ell$ and $\bar{z}$ are held fixed throughout the proof, we temporarily suppress them from the notation. Take two sequences $\bar{\pi}^2, \bar{\pi}^1 \in \mathcal{F}^\infty$, with $\bar{\pi}^2 \geq \bar{\pi}^1$, and let $\bar{\mu}^2 = B(\bar{\pi}^2)$ and $\bar{\mu}^1 = B(\bar{\pi}^1)$. We want to show that $\bar{\mu}^2 \geq \bar{\mu}^1$.Using the definition of $\hat{Q}_t$ in (22), $\bar{\mu}^2 \geq \bar{\mu}^1$ if and only if

$$\begin{aligned} &\int_X f(x') \int_X Q(x, 0; \mathrm{d}x')[\pi^2(\theta^{t-1})](\mathrm{d}x) + \int_X f(x') \int_X \hat{g}_t(x, \theta^t, \bar{\pi}^2)[Q(x, 1; \mathrm{d}x') - Q(x, 0; \mathrm{d}x')][\pi^2_t(\theta^{t-1})](\mathrm{d}x) \\ \geq &\int_X f(x') \int_X Q(x, 0; \mathrm{d}x')[\pi^1(\theta^{t-1})](\mathrm{d}x) + \int_X f(x') \int_X \hat{g}_t(x, \theta^t, \bar{\pi}^1)[Q(x, 1; \mathrm{d}x') - Q(x, 0; \mathrm{d}x')][\pi^1_t(\theta^{t-1})](\mathrm{d}x) \end{aligned}$$

for every increasing function $f : X \to \mathbb{R}$ and all $\theta^t \in \Theta^t$ and $t = 1, 2 \ldots$

Fix $t > 0$ and $\theta^t \in \Theta^t$, and focus on the first term in the sums above. Using Fubini's theorem to exchange the order of integration,

$$\int_X f(x') \int_X Q(x, 0; \mathrm{d}x')[\pi(\theta^{t-1})](\mathrm{d}x) = \int_X \left(\int_X f(x') Q(x, 0; \mathrm{d}x') \right) [\pi(\theta^{t-1})](\mathrm{d}x)$$

Under Assumption A.3, the term: $\int_X f(x')Q(x, 0; \mathrm{d}x')$ is an increasing function of $x \in X$. Thus

$$\int_X f(x') \int_X Q(x, 0; \mathrm{d}x')[\pi^2(\theta^{t-1})](\mathrm{d}x) \geq \int_X f(x') \int_X Q(x, 0; \mathrm{d}x')[\pi^1(\theta^{t-1})](\mathrm{d}x)$$

and we focus on the second term. We have

$$\begin{aligned} &\int_X f(x) \int_X \hat{g}_t(x, \theta^t, \bar{\pi})[Q(x, 1; \mathrm{d}x') - Q(x, 0; \mathrm{d}x')][\pi_t(\theta^{t-1})](\mathrm{d}x) \\ &= \int_X \hat{g}_t(x, \theta^t, \bar{\pi}) \left(\int_X f(x')[Q(x, 1; \mathrm{d}x') - Q(x, 0; \mathrm{d}x')] \right) [\pi_t(\theta^{t-1})](\mathrm{d}x) \end{aligned}$$

In this equation, the term $\hat{g}_t$ is greater than zero by construction and increasing in $x \in X$ by Corollary A.2. Under Assumption A.3, the term:

$$F(x) \equiv \int_X f(x')[Q(x, 1; \mathrm{d}x') - Q(x, 0; \mathrm{d}x')]$$

is also greater than zero and increasing in $x \in X$. Thus

$$\begin{aligned} \int_X \hat{g}_t(x, \theta^t, \bar{\pi}^2) F(x) [\pi_t^2(\theta^{t-1})](\mathrm{d}x') &\geq \int_X \hat{g}_t(x, \theta^t, \bar{\pi}^1) F(x) [\pi_t^2(\theta^{t-1})](\mathrm{d}x') \\ &\geq \int_X \hat{g}_t(x, \theta^t, \bar{\pi}^1) F(x) [\pi_t^1(\theta^{t-1})](\mathrm{d}x') \end{aligned}$$

because, by hypothesis, $\pi^2(\theta^{t-1})$ stochastically dominates $\pi^1(\theta^{t-1})$ for all $\theta^t \in \Theta^t$. Because the choice of t and θ^t was arbitrary, the above inequalities hold for all $t > 0$ and all element of Θ^t.

□

Let $\alpha : W^{\ell,\infty} \times Z^\infty \to \mathcal{F}^\infty$:

$$\alpha(\bar{w}^\ell, \bar{z}) = \{\bar{\pi} \in \mathcal{F}^\infty : \bar{\pi} \in B(\bar{\pi}, \bar{w}^\ell, \bar{z})\} \tag{24}$$

denote the set of fixed points of B. The main result of this subsection is the following proposition.

Proposition A.3. *Under Assumptions A.2-A.4, the fixed-point correspondence α is a non-empty complete lattice.*

Proof. The proof is based on Echenique (2005) (see especially his Section 4 for the continuous case). Because $\bar{w}^\ell$ and $\bar{z}$ are held fixed throughout the proof, we temporarily drop them from the notation. Let $\bar{\pi}^0 = \bigvee \mathcal{F}^\infty$ be the greatest element in $\mathcal{F}^\infty$; $\bar{\pi}^0$ is the sequence of mappings that assign measure one to the greatest element in X for all $\theta^t \in \Theta^t$ at all $t > 0$. Let $\{\bar{\pi}^n\}$ be the iterative sequence defined from: $\bar{\pi}^{n+1} = B(\bar{\pi}^n)$. Because $\bar{\pi}^1 \leq \bar{\pi}^0$ and B is increasing (Lemma A.3), $B(\bar{\pi}^1) \leq B(\bar{\pi}^0)$, and, by induction, $B(\bar{\pi}^{n+1}) \leq B(\bar{\pi}^n)$ for all $n > 0$. Therefore $\{\bar{\pi}^n\}$ is a decreasing sequence; because it is bounded from below, it converges to some element $\bar{\pi}^e$. The continuity of B (Lemma A.2) implies that $\bar{\pi}^e$ is a fixed point, because $\bar{\pi}^{2n+1} = B(\bar{\pi}^{2n})$, and both $\{\bar{\pi}^{2n+1}\}$ and $\{\bar{\pi}^{2n}\}$ converge to $\bar{\pi}^e$. Therefore α is nonempty. Further, suppose that $\bar{\pi}^{e'}$ is another fixed point in α. Then $\bar{\pi}^{e'} \leq \bar{\pi}^0$, and $\bar{\pi}^{e'} \leq \bar{\pi}^n$ implies: $\bar{\pi}^{e'} = B(\bar{\pi}^{e'}) \leq B(\bar{\pi}^{n+1}) = \bar{\pi}^n$. By induction, $\bar{\pi}^{e'} \leq \bar{\pi}^e$. Therefore $\bar{\pi}^e$ is the greatest element in α. Then, Theorem 2 in Echenique (2005) implies that α is a complete lattice. $\square$

A.8 Monotone Strategies for the Social Group

In this subsection, we demonstrate the monotonicity properties of the social group's correspondence α with respect to the history of leader's type $\bar{\theta}$ and the leader's plan $\bar{w}^\ell$. In both cases, we start by proving the properties for the optimal correspondence G_t and then show how they carry over to $\hat{g}_t$, B and, finally, the fixed-point set α. As a preliminary step, we show that the sequence of maps $(q_1, q_2, \dots)$ inherits the monotonicity properties of the leader's plans.

Lemma A.4. *Let $\bar{w}^{\ell,2}, \bar{w}^{\ell,1}$ be leader's plans in $W^{\ell,\infty}$, with $\bar{w}^{\ell,2} \geq \bar{w}^{\ell,1}$, and let $\bar{q}^1, \bar{q}^2$ be defined from* (15)*. Then, under Assumption A.4, $\bar{q}^2 \geq \bar{q}^1$.*

Proof. From the definitions (14) and (15), we have

$$[q(\theta^t)](A) = Q^\ell(\theta_t, 0; A) + \lambda(\{\varepsilon : w_t^\ell(\theta^t, \varepsilon) = 1\})[Q^\ell(\theta_t, 1; A) - Q^\ell(\theta_t, 0; A)].$$

Fix θ^t and t, and let $f : \Theta \to \mathbb{R}$ be an increasing integrable function; then

$$\begin{aligned}\int_\Theta f(\theta)[q(\theta^t)](\mathrm{d}\theta) = &\int_\Theta f(\theta) Q^\ell(\theta_t, 0; \mathrm{d}\theta) \\ &+ \lambda(\{\varepsilon : w_t^\ell(\theta^t, \varepsilon) = 1\}) \int_\Theta f(\theta) \left[Q^\ell(\theta_t, 1; \mathrm{d}\theta) - Q^\ell(\theta_t, 0; \mathrm{d}\theta)\right]\end{aligned}$$

The term $\int_\Theta f(\theta)\left[Q^\ell(\theta_t,1;\mathrm{d}\theta)-Q^\ell(\theta_t,0;\mathrm{d}\theta)\right]$ is positive because Q^ℓ is stochastically increasing. Therefore $q_t^2 \geq q_t^1$ because $\lambda(\{\varepsilon : w_t^{\ell,2}(\theta^t,\varepsilon)=1\}) > \lambda(\{\varepsilon : w_t^{\ell,1}(\theta^t,\varepsilon)=1\})$. Because the choice of θ^t and t was arbitrary, the inequality holds for all $\theta^t \in \Theta^t$ and all $t>0$.

□

Lemma A.5. *Suppose that a leader's plan $\bar{w}^\ell$ in $W^{\ell,\infty}$ is increasing in $\bar{\theta}$, and let $\bar{q}$ be defined from (15). Then, under Assumption A.4, $\bar{q}$ is also increasing in $\bar{\theta}$.*

Proof. From the definitions (14) and (15), we have

$$[q(\theta^t)](A) = Q^\ell(\theta_t,0;A) + \lambda(\{\varepsilon : w_t^\ell(\theta^t,\varepsilon)=1\})[Q^\ell(\theta_t,1;A)-Q^\ell(\theta_t,0;A)].$$

Fix θ^t and t, and let $f:\Theta\to\mathbb{R}$ be an increasing integrable function; then

$$\begin{aligned}\int_\Theta f(\theta)[q(\theta^t)](\mathrm{d}\theta) &= \int_\Theta f(\theta)Q^\ell(\theta_t,0;\mathrm{d}\theta)\\ &\quad + \lambda(\{\varepsilon : w_t^\ell(\theta^t,\varepsilon)=1\})\int_\Theta f(\theta)\left[Q^\ell(\theta_t,1;\mathrm{d}\theta)-Q^\ell(\theta_t,0;\mathrm{d}\theta)\right]\end{aligned}$$

First, the term $\int_\Theta f(\theta)Q^\ell(\theta_t,0;\mathrm{d}\theta)$ is increasing in θ^t because Q^ℓ is stochastically increasing. Second, the term $\int_\Theta f(\theta)\left[Q^\ell(\theta_t,1;\mathrm{d}\theta)-Q^\ell(\theta_t,0;\mathrm{d}\theta)\right]$ is positive and increasing in $\theta^t\in\Theta^t$ because Q^ℓ exhibits stochastically increasing differences. Therefore $q_t^2 \geq q_t^1$ because $\lambda(\{\varepsilon : w_t^{\ell,2}(\theta^t,\varepsilon)=1\}) > \lambda(\{\varepsilon : w_t^{\ell,1}(\theta^t,\varepsilon)=1\})$. Third, the term $\lambda(\{\varepsilon : w_t^\ell(\theta^t,\varepsilon)=1\})$ is increasing in $\theta^t\in\Theta^t$ because w_t^ℓ is increasing by assumption. Because the choice of θ^t and t was arbitrary, the inequality holds for all $\theta^t\in\Theta^t$ and all $t>0$.

□

A.8.1 With respect to θ

Proposition A.4. *Suppose that Assumptions A.2-A.4 hold. Moreover, suppose that $\bar{w}^\ell$ is increasing. If the payoff function exhibits increasing differences in (y,θ), (x,θ), and $\bar{\pi}$ is increasing, then the optimal policy correspondence in (18) is increasing in $\theta^t\in\Theta^t$ for all $t>0$. If the payoff function exhibits decreasing differences in (y,θ), (x,θ), and $\bar{\pi}$ is decreasing, then the optimal policy correspondence in (18) is decreasing in $\theta^t\in\Theta^t$ for all $t>0$.*

Proof. We only sketch the proof, since it is similar to the proof of Proposition A.8. For $\hat{V}_t^n$ with increasing differences in (x,θ^t), there we show that

$$\int_X \hat{V}^n_{t+1}(x',\theta^{t+1},\bar{w}^\ell)[Q(x,1;\mathrm{d}x')-Q(x,0;\mathrm{d}x')]$$

is increasing in $\theta^{t+1}\in\Theta^{t+1}$. Furthermore, by Lemma A.5 $\bar{q}$ is increasing if $\bar{w}^\ell$ is increasing. Then, for any $\theta^{1,t},\theta^{2,t}\in\Theta^t$ such that $\theta^{2,t}\geq\theta^{1,t}$

$$\begin{aligned}
&\int_\Theta\int_X \hat{V}^n_{t+1}(x',\theta_{t+1},\theta^{2,t},\bar{w}^\ell)[Q(x,1;\mathrm{d}x')-Q(x,0;\mathrm{d}x')][q_t(\theta^{2,t})](\mathrm{d}\theta_{t+1})\\
\geq&\int_\Theta\int_X \hat{V}^n_{t+1}(x',\theta_{t+1},\theta^{1,t},\bar{w}^\ell)[Q(x,1;\mathrm{d}x')-Q(x,0;\mathrm{d}x')][q_t(\theta^{2,t})](\mathrm{d}\theta_{t+1})\\
\geq&\int_\Theta\int_X \hat{V}^n_{t+1}(x',\theta_{t+1},\theta^{1,t},\bar{w}^\ell)[Q(x,1;\mathrm{d}x')-Q(x,0;\mathrm{d}x')][q_t(\theta^{1,t})](\mathrm{d}\theta_{t+1})
\end{aligned}$$

(note the slight abuse of notation in separating the arguments θ_{t+1} and θ^t). This shows that the integral term in (20) has increasing differences in (y,θ^t). Similarly, for $\hat{V}^n_t$ with decreasing differences in (x,θ^t)

$$\int_X \hat{V}^n_{t+1}(x',\theta^{t+1},\bar{w}^\ell)[Q(x,0;\mathrm{d}x')-Q(x,1;\mathrm{d}x')]$$

is increasing in $\theta^{t+1}\in\Theta^{t+1}$ and the integral term in (20) has decreasing differences in (y,θ^t). The rest of the proof follows the proof of Proposition A.8. □

We also consider a situation when the leader's type may affect the sequence of exogenous circumstances $\bar{z}\in Z^\infty$. That is, suppose that $z_t\in Z$ is not a real number but rather an increasing function $z_t:\Theta^t\to\mathbb{R}$.

Proposition A.5. *Let $z_t:\Theta^t\to\mathbb{R}$ be a function, and suppose that Assumptions A.2-A.4 hold. Moreover, suppose that $\bar{w}^\ell$ and $\bar{z}$ are increasing. If the payoff function exhibits increasing differences in (y,θ), (x,θ), (y,z) and (x,z), and $\bar{\pi}$ and $\bar{z}$ are increasing, then the optimal policy correspondence in (18) is increasing in $\theta^t\in\Theta^t$ for all $t>0$. If the payoff function exhibits decreasing differences in (y,θ), (x,θ), (y,z) and (x,z), $\bar{\pi}$ is decreasing and $\bar{z}$ is increasing, then the optimal policy correspondence in (18) is decreasing in $\theta^t\in\Theta^t$ for all $t>0$.*

Proof. We only sketch the proof. The properties of the integral term are not affected when z_t is a function, because z_t only enters the expression as a given sequence $\bar{z}\in Z^\infty$. Therefore we only have to check the properties of the payoff function. Let $\theta^{1,t},\theta^{2,t}\in\Theta^t$ such that $\theta^{2,t}\geq\theta^{1,t}$. First, suppose that u exhibits increasing differences in (y,θ), (x,θ), (y,z) and (x,z), and $\bar{\pi}$ and $\bar{z}$ are increasing. Then

$$
\begin{aligned}
&u(1,x,\theta_t^2,\pi_t(\theta^{2,t-1}),z_t(\theta^{2,t-1}))-u(0,x,\theta_t^2,\pi_t(\theta^{2,t-1}),z_t(\theta^{2,t-1}))\\
\geq &u(1,x,\theta_t^1,\pi_t(\theta^{2,t-1}),z_t(\theta^{2,t-1}))-u(0,x,\theta_t^1,\pi_t(\theta^{2,t-1}),z_t(\theta^{2,t-1}))\\
\geq &u(1,x,\theta_t^1,\pi_t(\theta^{1,t-1}),z_t(\theta^{2,t-1}))-u(0,x,\theta_t^1,\pi_t(\theta^{1,t-1}),z_t(\theta^{2,t-1}))\\
\geq &u(1,x,\theta_t^1,\pi_t(\theta^{1,t-1}),z_t(\theta^{1,t-1}))-u(0,x,\theta_t^2,\pi_t(\theta^{1,t-1}),z_t(\theta^{1,t-1}))
\end{aligned}
$$

Next, suppose that u exhibits decreasing differences in (y,θ), (x,θ), (y,z) and (x,z), and $\bar{\pi}$ is decreasing and $\bar{z}$ is increasing. Then all the previous inequalities are reversed.

□

Clearly, Proposition A.4 is a special case of Proposition A.5 for z_t constant over Θ^t at all $t>0$.

Corollary A.4. *Suppose that Assumption A.4 holds. If the optimal policy correspondence in (18) is increasing in θ^t, then $\hat{g}_t$ in (19) is also increasing in θ^t for all $t>0$. If the optimal policy correspondence in (18) is decreasing in θ^t, then $\hat{g}_t$ in (19) is also decreasing in θ^t for all $t>0$.*

The proof is omitted.

Lemma A.6. *Suppose that Assumptions A.2-A.4 hold. If the optimal policy correspondence in (18) is increasing in θ^t, and $\bar{\pi}$ is increasing, then $\bar{\pi}'=B(\bar{\pi},\bar{w}^\ell,\bar{z})$ is increasing. If the optimal policy correspondence in (18) is decreasing in θ^t, and $\bar{\pi}$ is decreasing, then $\bar{\pi}'=B(\bar{\pi},\bar{w}^\ell,\bar{z})$ is decreasing.*

Proof. Fix t, and write

$$
\begin{aligned}
\int_X f(x)\pi'_{t+1}(\theta^t)(\mathrm{d}x) &= \int_X f(x)\int_X \hat{Q}_t^n(x,\theta^t,\bar{\pi},\bar{w}^\ell;\mathrm{d}x)[\pi_t(\theta^{t-1})](\mathrm{d}x)\\
&= \int_X f(x)\int_X Q(x,0;\mathrm{d}x)[\pi_t(\theta^{t-1})](\mathrm{d}x)\\
&+ \int_X f(x)\int_X \hat{g}_t(x,\theta^t,\bar{\pi},\bar{w}^\ell)[Q(x,1;\mathrm{d}x)-Q(x,0;\mathrm{d}x)][\pi_t(\theta^{t-1})](\mathrm{d}x)\\
&= \int_X \left(\int_X f(x)Q(x,0;\mathrm{d}x)\right)[\pi_t(\theta^{t-1})](\mathrm{d}x)+\\
&\int_X \hat{g}_t(x,\theta^t,\bar{\pi},\bar{w}^\ell)\left(\int_X f(x')[Q(x,1;\mathrm{d}x')-Q(x,0;\mathrm{d}x')]\right)[\pi_t(\theta^{t-1})](\mathrm{d}x)
\end{aligned}
$$

where we have used Fubini's theorem to exchange the order of integration. If both G_t and π_t are increasing in θ^t, then all the terms in the sum above are increasing in θ^t. If both G_t and π_t are decreasing in θ^t, then all the terms in the sum above are decreasing in θ^t.

□

Proposition A.6. *Suppose that Assumptions A.1-A.4 hold. Moreover, suppose that $\bar{w}^\ell$ is increasing. If the payoff function exhibits increasing differences in (y, θ), (x, θ), then the greatest and the least elements in α are increasing with respect to $\bar{\theta} \in \Theta^\infty$. If the payoff function exhibits decreasing differences in (y, θ), (x, θ), then the greatest and the least elements in α are decreasing with respect to $\bar{\theta} \in \Theta^\infty$.*

Proof. The proof relies on the iterative argument of the proof of Proposition A.3. There we showed that, under Assumptions A.1-A.4, the greatest element of α can be found as the limit of the decreasing sequence $\bar{\pi}^{n+1} = B(\bar{\pi}^n)$, with $\bar{\pi}^0$ equal to the greatest element in F^∞. Recall that $\bar{\pi}^0$ is a sequence of mappings that assigns to all $\theta^t \in \Theta^t$ and all $t > 0$ a degenerate probability measure with mass one on the greatest element of X. So $\bar{\pi}^0$ is both increasing and decreasing with respect to $\bar{\theta}$. Suppose, first, that the payoff function has increasing differences in (y, θ) and (x, θ). With $\bar{w}^\ell$ increasing, Proposition A.4, Corollary A.4, and Lemma A.6 imply that $\bar{\pi}^{n+1} = B(\bar{\pi}^n)$ is increasing whenever $\bar{\pi}^n$ is increasing. By induction, we find that the greatest element in α is increasing with respect to $\bar{\theta}$. Next, suppose that the payoff function has decreasing differences in (y, θ) and (x, θ). With $\bar{w}^\ell$ increasing, Proposition A.4, Corollary A.4, and Lemma A.6 imply that $\bar{\pi}^{n+1} = B(\bar{\pi}^n)$ is decreasing whenever $\bar{\pi}^n$ is decreasing. By induction, the greatest element in α is decreasing with respect to $\bar{\theta}$. A symmetric argument holds for the least element. □

We also deal with the case where z_t is an increasing function analyzed in Proposition A.5.

Proposition A.7. *Let $z_t : \Theta^t \to \mathbb{R}$ be a function, and suppose that Assumptions A.2-A.4 hold. Moreover, suppose that $\bar{w}^\ell$ and $\bar{z}$ are increasing. If the payoff function exhibits increasing differences in (y, θ), (x, θ), (y, z) and (x, z), then the greatest and the least elements in α are increasing with respect to $\bar{\theta} \in \Theta^\infty$. If the payoff function exhibits decreasing differences in (y, θ), (x, θ), (y, z) and (x, z), then the greatest and the least elements in α are decreasing with respect to $\bar{\theta} \in \Theta^\infty$.*

The proof parallels the proof of Proposition A.6, except that Proposition A.5 needs to be used to characterize the properties of the optimal policy correspondence.

A.8.2 With respect to $\bar{w}^\ell$

Proposition A.8. *Suppose that Assumptions A.2-A.4 hold. If the payoff function exhibits increasing differences in (y, θ), (x, θ), and $\bar{\pi}$ is increasing, then the optimal policy correspondence in (18)* *is increasing in $\bar{w}^\ell$. If the payoff function exhibits decreasing differences in (y, θ), (x, θ), and $\bar{\pi}$ is decreasing, then the optimal policy correspondence in (18) is decreasing in $\bar{w}^\ell$.*

Proof. We follow the same logic of the proof of Proposition A.2. Because $\bar{\pi}$ and $\bar{z}$ are held constant throughout the proof, we temporarily drop them from the notation.

Start from some $\hat{V}^{\infty,n}$ such that each $\hat{V}_t^n$ exhibits increasing differences in (x, θ^{t+1}). For $\theta^t \in \Theta^t$ given and any $\theta_{t+1}^1, \theta_{t+1}^2 \in \Theta$ such that $\theta_{t+1}^2 \geq \theta_{t+1}^1$, this implies that $\hat{V}_{t+1}^n(x, \theta^{2,t+1}, \bar{w}^\ell) - \hat{V}_{t+1}^n(x, \theta^{1,t+1}, \bar{w}^\ell)$ is an increasing function of $x \in X$, and that

$$\int_X \left[\hat{V}_{t+1}^n(x', \theta^{2,t+1}, \bar{w}^\ell) - \hat{V}_{t+1}^n(x', \theta^{1,t+1}, \bar{w}^\ell)\right] Q(x, y; \mathrm{d}x')$$

is increasing in $y \in \{0, 1\}$, because Q is stochastically increasing. Rearranging terms, we find that

$$\int_X \hat{V}_{t+1}^n(x', \theta^{t+1}, \bar{w}^\ell)[Q(x, 1; \mathrm{d}x') - Q(x, 0; \mathrm{d}x')]$$

is increasing in $\theta_{t+1} \in \Theta$. Furthermore, suppose that each $\hat{V}_t^n$ in $\hat{V}^{\infty,n}$ exhibits increasing differences in $(x, \bar{w}^\ell)$. By the same argument, the term above is an increasing function of $\bar{w}^\ell \in W^{\ell,\infty}$.

Now consider $\bar{w}^{\ell,1}, \bar{w}^{\ell,2} \in W^{\ell,\infty}$ such that $\bar{w}^{\ell,2} \geq \bar{w}^{\ell,1}$, and let $\bar{q}^1, \bar{q}^2$ be the associated mappings according to (15). Combining the previous results, we obtain

$$\begin{aligned}
&\int_\Theta \int_X \hat{V}_{t+1}^n(x', \theta^{t+1}, \bar{w}^{\ell,2}) \left[Q(x, 1; \mathrm{d}x') - Q(x, 0; \mathrm{d}x')\right] [q_t^2(\theta^t)](\mathrm{d}\theta_{t+1}) \\
\geq &\int_\Theta \int_X \hat{V}_{t+1}^n(x', \theta^{t+1}, \bar{w}^{\ell,1}) \left[Q(x, 1; \mathrm{d}x') - Q(x, 0; \mathrm{d}x')\right] [q_t^2(\theta^t)](\mathrm{d}\theta_{t+1}) \\
\geq &\int_\Theta \int_X \hat{V}_{t+1}^n(x', \theta^{t+1}, \bar{w}^{\ell,1}) \left[Q(x, 1; \mathrm{d}x') - Q(x, 0; \mathrm{d}x')\right] [q_t^1(\theta^t)](\mathrm{d}\theta_{t+1})
\end{aligned}$$

because by Lemma A.4 $q^2 \geq q^1$ for all $t > 0$. Thus we have shown that

$$\int_\Theta \int_X \hat{V}_{t+1}^n(x', \theta^{t+1}, \bar{w}^\ell) Q(x, y; \mathrm{d}x')[q_t(\theta^t)](\mathrm{d}\theta_{t+1})$$

has increasing differences in $(y, \bar{w}^\ell)$. To apply the argument recursively, we have to verify that the assumptions carry over to the next element of the iteration, $\hat{V}_t^{n+1}$. If π_t is increasing with respect to θ^t, this is the case because: (1) u has increasing differences in (x, p) and in (x, θ); (2) u does not depend on $\bar{w}^\ell$. Therefore all the elements of the sequence $\hat{V}^{\infty,n}$ exhibit increasing differences in $(y, \bar{w}^\ell)$ and so does the pointwise limit $\hat{V}^\infty$. The result follows from the definition of G_t in (18) and from Theorem 2.8.4 in Topkis (1998). □

The next corollary is an immediate consequence of this Proposition.

Corollary A.5. *Suppose that Assumption A.4 holds. If the optimal policy correspondence in (18) is increasing in $\bar{w}^\ell$, then $\hat{g}_t$ in (19) is also increasing in $\bar{w}^\ell$ for all $t > 0$. If the optimal policy correspondence in (18) is decreasing in $\bar{w}^\ell$, then $\hat{g}_t$ in (19) is also decreasing in $\bar{w}^\ell$ for all $t > 0$.*

The proof is omitted.

Lemma A.7. *Suppose that Assumption A.4 holds. If the optimal policy correspondence in (18) is increasing in $\bar{w}^\ell$, then B is increasing in $\bar{w}^\ell$. If the optimal policy correspondence in (18) is decreasing in $\bar{w}^\ell$, then B is decreasing in $\bar{w}^\ell$.*

In Lemma A.3 we proved that B is increasing in $\bar{\pi}$ using the fact that G_t is increasing in $\bar{\pi}$ (see Proposition A.2). Lemma A.7 also follows from the properties of G_t with respect to $\bar{w}^\ell$ (see Proposition A.8).

Proposition A.9. *Suppose that Assumptions A.1-A.4 hold, and let $\tilde{W}^{\ell,\infty} \subset W^{\ell,\infty}$ be the subset of increasing leader's plans. If the payoff function exhibits increasing differences in (y, θ) and (x, θ), then the greatest and the least elements of α are increasing in $\bar{w}^\ell \in \tilde{W}^{\ell,\infty}$. If the payoff function exhibits decreasing differences in (y, θ) and (x, θ), then the greatest and the least elements of α are decreasing in $\bar{w}^\ell \in \tilde{W}^{\ell,\infty}$.*

Proof. The proof relies on the iterative argument of the proof of Proposition A.3. Start from the greatest element in $\mathcal{F}^\infty$, $\bar{\pi}^0$, i.e. the sequence of mappings that assign probability one to the greatest element in X, for all $t > 0$ and all $\theta^t \in \Theta^t$. Note that $\bar{\pi}^0$ is both increasing and decreasing. Let $\bar{w}^{\ell,1}$ and $\bar{w}^{\ell,2}$ in $\tilde{W}^{\ell,\infty}$ with $\bar{w}^{\ell,2} \geq \bar{w}^{\ell,1}$, and let $\bar{\pi}^{2,n+1} = B(\bar{\pi}^{2,n}, \bar{w}^{\ell,2})$ and $\bar{\pi}^{1,n+1} = B(\bar{\pi}^{1,n}, \bar{w}^{\ell,1})$, with $\bar{\pi}^{2,0} = \bar{\pi}^{1,0} = \bar{\pi}^0$. Suppose, first, that the payoff function has increasing differences in (y, θ) and (x, θ).

Because $\bar{\pi}^0$ is increasing, Lemma A.7 implies that $\bar{\pi}^{2,1} = B(\bar{\pi}^0, \bar{w}^{\ell,2}) \geq B(\bar{\pi}^0, \bar{w}^{\ell,1}) = \bar{\pi}^{1,1}$. Further, because $\bar{w}^{\ell,1}$ and $\bar{w}^{\ell,2}$ are, by construction, increasing in $\bar{\theta}$, Lemma A.6 implies that $\bar{\pi}^{2,1}$ and $\bar{\pi}^{1,1}$ are also increasing in $\bar{\theta}$. Then, because B is increasing in $\bar{\pi}$ (Lemma A.3), we have $B(\bar{\pi}^{2,1}, \bar{w}^{\ell,2}) \geq B(\bar{\pi}^{2,1}, \bar{w}^{\ell,1}) \geq B(\bar{\pi}^{1,1}, \bar{w}^{\ell,1})$. By induction, $\bar{\pi}^{2,n+1} = B(\bar{\pi}^{2,n}, \bar{w}^{\ell,2}) \geq B(\bar{\pi}^{2,n}, \bar{w}^{\ell,1}) \geq B(\bar{\pi}^{1,n}, \bar{w}^{\ell,1}) = \bar{\pi}^{1,n+1}$ for all $n > 0$. By Proposition A.3, the limits $\bar{\pi}^{2,e} = \lim_{n\to\infty} \bar{\pi}^{2,n}$ and $\bar{\pi}^{1,e} = \lim_{n\to\infty} \bar{\pi}^{1,n}$ are, respectively, the greatest elements in $\alpha(\bar{w}^{\ell,2})$ and $\alpha(\bar{w}^{\ell,1})$, and $\bar{\pi}^{2,e} \geq \bar{\pi}^{1,e}$.

Next, suppose that the payoff function has decreasing differences in (y, θ) and (x, θ). Because $\bar{\pi}^0$ is decreasing, Lemma A.6 implies that $\bar{\pi}^1 = B(\bar{\pi}^0, \bar{w}^\ell)$ decreases with $\bar{\theta}$ for each $\bar{w}^\ell \in \tilde{W}^{\ell,\infty}$, and Lemma A.7 implies that B decreases with $\bar{w}^\ell \in \tilde{W}^{\ell,\infty}$. Because B is increasing in $\bar{\pi}$ (Lemma A.3), we have $\bar{\pi}^{2,n+1} = B(\bar{\pi}^{2,n}, \bar{w}^{\ell,2}) \leq B(\bar{\pi}^{2,n}, \bar{w}^{\ell,1}) \leq B(\bar{\pi}^{1,n}, \bar{w}^{\ell,1}) = \bar{\pi}^{1,n+1}$ for all $n > 0$. A symmetric argument holds for the least element. $\square$

A.9 The Leader's Problem

Let $u^\ell : \{0,1\} \times \Theta \times \mathcal{P} \times \mathcal{Z}$ be defined from $u^\ell(y, \theta, p, z) = u(y, \theta, \theta, p, z)$; then, the leader's optimal nonstationary policy correspondence $G_t^\ell : \Theta^t \times \mathcal{E} \times \mathcal{F}^\infty \times Z^\infty \to \{0,1\}$ is defined from:

$$G_t^\ell(\theta^t, \varepsilon, \bar{\pi}, \bar{z}) = \underset{y \in \{0,1\}}{\arg\max}\, u^\ell(y, \theta_t, \pi_t(\theta^{t-1}), z_t) + \varepsilon(y) + \beta \int_\Theta \hat{V}_{t+1}^\ell(\theta^{t+1}, \bar{\pi}, \bar{z}) Q^\ell(\theta_t, y; \mathrm{d}\theta_{t+1}). \tag{25}$$

There are two main differences with respect to Equation (18): (1) the leader's uncertainty is confined to Q^ℓ, because θ is her individual type; (2) her individual type θ also enters into π_t: thus the leader understands that her decisions will affect the aggregate behavior of the social group.

$$\hat{g}_t^\ell\left(\theta^t, \bar{\pi}, \bar{z}\right) = \lambda\Big(\{\varepsilon \in \mathcal{E} : g_t^\ell(\theta^t, \varepsilon, \bar{\pi}, \bar{z}) = 1\}\Big) \quad \text{all } t = 1, 2 \ldots \tag{26}$$

Lemmas A.1 applies to (25).

For a leader's plan $\bar{w}^\ell$ to be optimal, it must agree with (25). Formally, we can define the leader's best response correspondence $\alpha^\ell : \mathcal{F}^\infty \times Z^\infty \to W^{\ell,\infty}$, as follows:

$$\alpha^\ell(\bar{\pi}, \bar{z}) = \{\bar{w}^\ell \in W^{\ell,\infty} : w_t^\ell(\theta^t, \varepsilon) \in G_t^\ell(\theta^t, \varepsilon, \bar{\pi}, \bar{z}) \text{ all } \theta^t \in \Theta^t, \varepsilon \in \mathcal{E}, t = 1, 2, \ldots\}. \tag{27}$$

A.10 Monotone Strategies for the Leader

In this subsection, we demonstrate the monotonicity properties of the leader's optimal behavior. We will need the following additional assumption:

Assumption A.5. *The leader's payoff function u^ℓ is weakly increasing in $p \in \mathcal{P}$:*

We then obtain the following proposition.

Proposition A.10. *Suppose that Assumptions A.1-A.5 hold. Then the optimal policy correspondence in (25) is increasing in $\bar{\pi}$. Moreover, it is increasing in $\theta^t \in \Theta^t$ for all $t > 0$ if $\bar{\pi}$ is increasing* *in θ^t. In particular, these properties hold for the greatest and least elements of the optimal policy correspondence.*

Proof. The proof with respect to $\bar{\pi}$ is analogous to the proof that G_t is increasing in $\bar{\pi}$ (see Proposition A.2). By contrast, the proof with respect to θ^t is more complicated because the leader's type also enters into the sequence $\bar{\pi}$. We only report the proof for the latter case. Because $\bar{z}$ is held fixed throughout the proof, we temporarily drop it from the notation.

The sequence of ex ante value functions for the leader $\hat{V}^{\ell,\infty}$ can be found as the limit of the sequence $\{\hat{V}^{\ell,\infty,n}\}$ in $\hat{\mathcal{V}}^{\ell,\infty}$ defined from:

$$\begin{aligned}\hat{V}_t^{\ell,n+1}(\theta_t,\theta^{t-1},\bar{\pi}) = \int_{\mathcal{E}}\Bigg\{ &\max_{d\in\{0,1\}} u^{\ell}(y,\theta_t,\pi_t(\theta^{t-1})) + \varepsilon(y) \\ &+ \beta\int_{\Theta}\hat{V}_{t+1}^{\ell,n}(\theta_{t+1},\theta^t,\bar{\pi})Q^{\ell}(\theta_t,y;\mathrm{d}\theta_{t+1})\Bigg\}\lambda(\mathrm{d}\varepsilon)\end{aligned} \tag{28}$$

where each element of the sequence converges pointwise.

Start from an arbitrary sequence $\hat{V}^{\ell,\infty,n} = \{\hat{V}_t^{\ell,n}\}_{t=1}^{\infty}$ such that each $\hat{V}_t^{\ell,n}$ is increasing in $\theta^t \in \Theta^t$ for all $t > 0$. Assumption A.3 implies that for all $\theta_t^1, \theta_t^2 \in \Theta$ such that $\theta_t^2 \geq \theta_t^1$:

$$\begin{aligned}&\int_{\Theta}\hat{V}_{t+1}^{\ell,n}(\theta_{t+1},\theta^t,\bar{\pi})Q^{\ell}(\theta_t^2,1;\mathrm{d}\theta_{t+1}) - \int_{\Theta}\hat{V}_{t+1}^{\ell,n}(\theta_{t+1},\theta^t,\bar{\pi})Q^{\ell}(\theta_t^2,0;\mathrm{d}\theta_{t+1}) \\ \geq &\int_{\Theta}\hat{V}_{t+1}^{\ell,n}(\theta_{t+1},\theta^t,\bar{\pi})Q^{\ell}(\theta_t^1,1;\mathrm{d}\theta_{t+1}) - \int_{\Theta}\hat{V}_{t+1}^{\ell,n}(\theta_{t+1},\theta^t,\bar{\pi})Q^{\ell}(\theta_t^1,0;\mathrm{d}\theta_{t+1})\end{aligned}$$

for all $\theta^t \in \Theta^t$. Note that we are keeping θ^t fixed inside the value function.

Second, suppose that each $\hat{V}_{t+1}^{\ell,n}$ is supermodular in $\theta^{t+1} \in \Theta^{t+1}$ for all t, so that

$$\hat{V}_{t+1}^{\ell,n}(\theta_{t+1},\theta^{2,t},\bar{\pi}) - \hat{V}_{t+1}^{\ell,n}(\theta_{t+1},\theta^{1,t},\bar{\pi})$$

is an increasing function of $\theta_{t+1} \in \Theta$. Then Assumption A.3 implies that

$$\begin{aligned}&\int_{\Theta}\hat{V}_{t+1}^{\ell,n}(\theta_{t+1},\theta^{2,t},\bar{\pi})Q^{\ell}(\theta_t,1;\mathrm{d}\theta_{t+1}) - \int_{\Theta}\hat{V}_{t+1}^{\ell,n}(\theta_{t+1},\theta^{2,t},\bar{\pi})Q^{\ell}(\theta_t,0;\mathrm{d}\theta_{t+1}) \\ \geq &\int_{\Theta}\hat{V}_{t+1}^{\ell,n}(\theta_{t+1},\theta^{1,t},\bar{\pi})Q^{\ell}(\theta_t,1;\mathrm{d}\theta_{t+1}) - \int_{\Theta}\hat{V}_{t+1}^{\ell,n}(\theta_{t+1},\theta^{1,t},\bar{\pi})Q^{\ell}(\theta_t,0;\mathrm{d}\theta_{t+1})\end{aligned}$$

for all $\theta_t \in \Theta$. Note that now we are keeping θ_t fixed inside the transition kernel. Combining the two previous observations, we obtain:

$$\begin{aligned}
&\int_\Theta \hat{V}^{\ell,n}_{t+1}(\theta_{t+1},\theta^{2,t},\bar{\pi})Q^\ell(\theta^2_t,1;\mathrm{d}\theta_{t+1}) - \int_\Theta \hat{V}^{\ell,n}_{t+1}(\theta_{t+1},\theta^{2,t},\bar{\pi})Q^\ell(\theta^2_t,0;\mathrm{d}\theta_{t+1}) \\
\geq &\int_\Theta \hat{V}^{\ell,n}_{t+1}(\theta_{t+1},\theta^{2,t},\bar{\pi})Q^\ell(\theta^1_t,1;\mathrm{d}\theta_{t+1}) - \int_\Theta \hat{V}^{\ell,n}_{t+1}(\theta_{t+1},\theta^{2,t},\bar{\pi})Q^\ell(\theta^1_t,0;\mathrm{d}\theta_{t+1}) \\
\geq &\int_\Theta \hat{V}^{\ell,n}_{t+1}(\theta_{t+1},\theta^{1,t},\bar{\pi})Q^\ell(\theta^1_t,1;\mathrm{d}\theta_{t+1}) - \int_\Theta \hat{V}^{\ell,n}_{t+1}(\theta_{t+1},\theta^{1,t},\bar{\pi})Q^\ell(\theta^1_t,0;\mathrm{d}\theta_{t+1})
\end{aligned}$$

Thus we have proven that the term:

$$\int_\Theta \hat{V}^{\ell,n}_{t+1}(\theta_{t+1},\theta^{t},\bar{\pi})Q^\ell(\theta_t,y;\mathrm{d}\theta_{t+1})$$

exhibits increasing differences in (y,θ^t). Because the utility function u^ℓ is supermodular, we conclude that the maximand in (28) also exhibits increasing differences in (y,θ^t).

Next, we have to check that the two properties assumed to obtain this result are indeed passed on to the next element of the sequence $\{\hat{V}^{\ell,\infty,n}\}_n$. First, we verify that $\hat{V}^{\ell,n}_t$ is increasing in θ_t. If $\bar{\pi}$ is increasing, then u^ℓ is increasing in θ^t because it is increasing in both its θ and p arguments. Furthermore, because Q^ℓ is stochastically supermodular and $\hat{V}^{\ell,n}_{t+1}$ is, by construction, increasing in θ_{t+1}, we have

$$\begin{aligned}
\int_\Theta \hat{V}^{\ell,n}_{t+1}(\theta_{t+1},\theta^{2,t},\bar{\pi})Q^\ell(\theta_{2,t},y;\mathrm{d}\theta_{t+1}) &\geq \int_\Theta \hat{V}^{\ell,n}_{t+1}(\theta_{t+1},\theta^{2,t},\bar{\pi})Q^\ell(\theta_{1,t},y;\mathrm{d}\theta_{t+1}) \\
&\geq \int_\Theta \hat{V}^{\ell,n}_{t+1}(\theta_{t+1},\theta^{1,t},\bar{\pi})Q^\ell(\theta_{1,t},y;\mathrm{d}\theta_{t+1})
\end{aligned}$$

for each $y \in \{0,1\}$. It follows that $\hat{V}^{\ell,n+1}_t$ is increasing in θ_t.

Second, we verify that $\hat{V}^{\ell,n}_t$ is supermodular in $\theta^t \in \Theta^t$. If $\bar{\pi}$ is increasing with respect to $\bar{\theta}$, then u^ℓ is supermodular. The supermodularity of $\hat{V}^{\ell,n}_{t+1}$ is preserved under the integral sign. Then the maximand is supermodular because it is the sum of supermodular functions. Finally, $\hat{V}^{\ell,n}_t$ is supermodular because supermodularity is preserved by the max operator and, again, the integral operator.

Because these results hold for an arbitrary time index t, we conclude that each $\hat{V}^{\ell,n}_t$ in $\hat{V}^{\ell,\infty,n}$ is increasing and supermodular in θ^t for $t > 0$. Applying the argument recursively, the property must hold for all elements of the sequence $\{\hat{V}^{\ell,\infty,n}\}_{n=1}^\infty$.

Now define G^ℓ_t as in (25). By the argument above, the maximand has strictly increasing differences in (y,θ^t). Hence by Theorem 2.8.3 in Topkis (1998), G^ℓ_t is increasing in θ^t. In particular, G^ℓ_t

has a greatest and least elements for all $\theta^t \in \Theta^t$, and they are increasing in θ^t. $\square$

A.11 Equilibrium with a Leader

Definition 2. An equilibrium with a leader is a pair $\bar{\pi}, \bar{w}^\ell \in \mathcal{F}^\infty \times W^{\ell,\infty}$ such that:

$$\bar{\pi} \in \alpha(\bar{w}^\ell, \bar{z}), \text{ and } \bar{w}^\ell \in \alpha^\ell(\bar{\pi}, \bar{z})$$

Let $E^\ell(\bar{z})$ denote the (possibly empty) set of equilibria with a leader.

Theorem A.1. *Suppose that Assumptions A.1-A.4. Moreover, suppose that the payoff function has increasing differences in (y, θ) and (x, θ). Then the set of equilibria with a leader, E^ℓ, is nonempty and has a greatest and a least element.*

Proof. We follow the logic of the proof of Lemma 6 in Van Zandt and Vives (2007). Since $\bar{z}$ is held fixed throughout, we temporarily drop it from the notation. Proposition A.3 implies that α has a greatest and a least element, and Proposition A.10 implies that α^ℓ has a greatest and a least element. Let $\alpha_\vee(\bar{w}^\ell)$ denote the greatest element in $\alpha(\bar{w}^\ell)$ for each $\bar{w}^\ell \in W^{\ell,\infty}$, and let $\alpha^\ell_\vee(\bar{\pi})$ denote the greatest element in $\alpha^\ell(\bar{\pi})$ for each $\bar{\pi} \in F^\infty$. Similarly, let $\alpha_\wedge(\bar{w}^\ell)$ and $\alpha^\ell_\wedge(\bar{\pi})$ denote the least elements.

Consider the iterative sequence $\{\bar{\pi}^n, \bar{w}^{\ell,n}\}$ defined from $\bar{\pi}^{n+1} = \alpha_\vee(\bar{w}^{\ell,n})$ and $\bar{w}^{\ell,n+1} = \alpha^\ell_\vee(\bar{\pi}^n)$, with $\bar{\pi}^0$ and $\bar{w}^{\ell,0}$ equal to the greatest elements in $\mathcal{F}^\infty$ and in $W^{\ell,\infty}$, respectively. By construction, $\bar{\pi}^0$, $\bar{w}^{\ell,0}$ are increasing with $\bar{\pi}^1 \leq \bar{\pi}^0$ and $\bar{w}^{\ell,1} \leq \bar{w}^{\ell,0}$. We now show that these properties imply that $\bar{\pi}^1$, $\bar{w}^{\ell,1}$ are increasing with $\alpha^\ell_\vee(\bar{\pi}^1) \leq \alpha^\ell_\vee(\bar{\pi}^0)$ and $\alpha_\vee(\bar{w}^{\ell,1}) \leq \alpha_\vee(\bar{w}^{\ell,0})$. First, because $\bar{\pi}^0$ and $\bar{w}^{\ell,0}$ are increasing, Propositions A.6 and A.10 imply, respectively, that $\bar{\pi}^1$ and $\bar{w}^{\ell,1}$ are increasing. Second, by Proposition A.10 $\bar{\pi}^1 \leq \bar{\pi}^0$ immediately implies $\alpha^\ell_\vee(\bar{\pi}^1) \leq \alpha^\ell_\vee(\bar{\pi}^0)$. Third, because both $\bar{w}^{\ell,0}$ and $\bar{w}^{\ell,1}$ are increasing, Proposition A.9 implies $\alpha_\vee(\bar{w}^{\ell,1}) \leq \alpha_\vee(\bar{w}^{\ell,0})$. By induction, $\{\bar{\pi}^n, \bar{w}^{\ell,n}\}$ is a decreasing sequence in $\mathcal{F}^\infty \times W^{\ell,\infty}$ and therefore converges to some element $(\bar{\pi}^e, \bar{w}^{\ell,e})$. Because $\bar{\pi}^n$ is a decreasing sequence and G^ℓ_t is an upper-hemicontinuous and increasing correspondence, we have $\bar{w}^{\ell,e} = \lim_{n\to\infty} \alpha^\ell_\vee(\bar{\pi}^n) = \alpha^\ell_\vee(\bar{\pi}^e)$. The continuity of α with respect to $\bar{w}^\ell$ follows from the continuity of B, so that $\bar{\pi}^e = \lim_{n\to\infty} \alpha_\vee(\bar{w}^{\ell,n}) = \alpha_\vee(\bar{w}^{\ell,e})$. Therefore $(\bar{\pi}^e, \bar{w}^{\ell,e})$ is an equilibrium with a leader. Further, suppose that $(\bar{\pi}^{e'}, \bar{w}^{\ell,e'})$ is another equilibrium. Then $(\bar{\pi}^{e'}, \bar{w}^{\ell,e'}) \leq (\bar{\pi}^0, \bar{w}^{\ell,0})$, and $(\bar{\pi}^{e'}, \bar{w}^{\ell,e'}) \leq (\bar{\pi}^n, \bar{w}^{\ell,n})$ implies $(\bar{\pi}^{e'}, \bar{w}^{\ell,e'}) = (\alpha_\vee(\bar{w}^{\ell,e'}), \alpha^\ell_\vee(\bar{\pi}^{e'})) \leq (\alpha_\vee(\bar{w}^{\ell,n}), \alpha^\ell_\vee(\bar{\pi}^n) \leq (\bar{\pi}^{n+1}, \bar{w}^{\ell,n+1})$. By induction, $(\bar{\pi}^{e'}, \bar{w}^{\ell,e'}) \leq (\bar{\pi}^e, \bar{w}^{\ell,e})$. Therefore $(\bar{\pi}^e, \bar{w}^{\ell,e})$ is the greatest element in E^ℓ. A symmetric argument holds for the least element in E^ℓ. $\square$

Proposition A.11. *Suppose that Assumptions A.1-A.4. Moreover, suppose that the payoff function has increasing differences in* (y, θ) *and* (x, θ)*. Then, the greatest and the least equilibria with a leader are increasing with respect to* $\bar{\theta} \in \Theta^{\infty}$*.*

Proof. We only sketch the proof as it follows the same iterative argument of the proof of Theorem A.1. In Proposition A.6 we showed that if $\bar{w}^{\ell}$ is increasing with respect to $\bar{\theta}$, then the greatest element in $\alpha(\bar{w}^{\ell})$ is increasing with respect to $\bar{\theta}$, and in Proposition A.10 we showed that if $\bar{\pi}$ is increasing with respect to $\bar{\theta}$, then the greatest element in $\alpha^{\ell}(\bar{\pi})$ is increasing with respect to $\bar{\theta}$. Given that $(\bar{\pi}^0, \bar{w}^{\ell,0})$ in the proof of Theorem A.1 are increasing with respect to $\bar{\theta}$, the property holds along the sequence $(\bar{\pi}^n, \bar{w}^{\ell,n})$ and for the limit $(\bar{\pi}^e, \bar{w}^{\ell,e})$, which is the greatest element of E^{ℓ}. A symmetric argument holds for the least element.

□

The fixed-point correspondences α and α^{ℓ}, together, define an increasing correspondence that maps a complete lattice into itself. While $\alpha(\bar{w}^{\ell})$ and $\alpha^{\ell}(\bar{\pi})$ are complete lattices for each $\bar{w}^{\ell}$ and $\bar{\pi}$, respectively, they are not necessarily subcomplete sublattices of their respective sets.

A.12 Comparative statics with respect to $\bar{z}$

In this subsection, we investigate the comparative statics of the equilibrium set with respect to unanticipated changes to the sequence of exogenous circumstances $\bar{z}$. We start by characterizing the properties of the optimal policy correspondence G_t and then show how they carry over to $\hat{g}_t$, B, and α. Then, we characterize the properties of the leader's optimal correspondence G_t^{ℓ}. Finally, we derive the implications for the set of equilibria with a leader.

Proposition A.12. *Suppose that Assumption A.4 hold. If the payoff function exhibits increasing differences in* (y, z) *and* (x, z)*, then the optimal policy correspondence in* (18) *is increasing in* $\bar{z}$ *for all* $t > 0$*. If the payoff function exhibits decreasing differences in* (y, z) *and* (x, z)*, then the optimal policy correspondence in* (18) *is decreasing in* $\bar{z}$ *for all* $t > 0$*.*

The proof of Proposition A.2 shows that G_t is increasing in $\bar{\pi}$. For the case with increasing differences, the proof of Proposition A.12 follows the same steps with $\bar{\pi}$ replaced with $\bar{z}$ (the proof for the case with decreasing differences is analogous).

Corollary A.6. *Suppose that Assumption A.4 holds. If the optimal policy correspondence in* (18) *is increasing in* $\bar{z}$*, then* $\hat{g}_t$ *in* (19) *is also increasing in* $\bar{z}$ *for all* $t > 0$*. If the optimal policy correspondence in* (18) *is decreasing in* $\bar{z}$*, then* $\hat{g}_t$ *in* (19) *is also decreasing in* $\bar{z}$ *for all* $t > 0$*.*

The proof is omitted.

Lemma A.8. *If the optimal policy correspondence in* (18) *is increasing in* $\bar{z}$*, then* B *is increasing in* $\bar{z}$*. If the optimal policy correspondence in* (18) *is decreasing in* $\bar{z}$*, then* B *is decreasing in* $\bar{z}$*.*

In Lemma A.3 we proved that B is increasing in $\bar{\pi}$ using the fact that G_t is increasing in $\bar{\pi}$ (see Proposition A.2). Lemma A.8 also follows from the properties of G_t with respect to $\bar{z}$ (see Proposition A.12). Because $\bar{\pi}$ is held fixed, so far in this section we did not restrict the way it enters the payoff function - i.e., we did not impose Assumption A.2. From now on we need to impose Assumption A.3 to guarantee that α is nonempty.

Proposition A.13. *Suppose that Assumptions A.1-A.4 hold. If the payoff function exhibits increasing differences in* (y, z) *and* (x, z)*, then the greatest and the least elements of* α *in* (24) *are increasing in* $\bar{z} \in Z^{\infty}$*. If the payoff function exhibits decreasing differences in* (y, z) *and* (x, z)*, then the greatest and the least elements of* α *in* (24) *are decreasing in* $\bar{z} \in Z^{\infty}$*.*

In Proposition A.9, we showed that the greatest and least elements of α may be increasing or decreasing in $\bar{w}^{\ell}$ depending on the properties B with respect to $\bar{w}^{\ell}$ (see Lemma A.7). Similarly, the proof of Proposition A.13 follows from the properties of B with respect to $\bar{z}$ (see Lemma A.8).

The next proposition is essentially a restatement of Proposition A.12, except that it refers to the leader rather than to the ordinary agents. However, we report it here for convenience.

Proposition A.14. *Suppose that Assumption A.4 hold. If the payoff function exhibits increasing differences in* (y, z) *and* (x, z)*, then the leader's optimal policy correspondence in* (25) *is increasing in* $\bar{z}$ *for all* $t > 0$*.*

Proposition A.15. *Suppose that Assumptions A.1-A.4 hold. If the payoff function exhibits increasing differences in* (y, z) *and* (x, z)*, then then the greatest and the least equilibria with a leader are increasing in* $\bar{z} \in Z^{\infty}$*.*

Proof. We only sketch the proof, as it follows the iterative argument in the proof of Theorem A.1. Consider $\bar{z}^1, \bar{z}^2 \in Z^{\infty}$. The proof of Theorem A.1 shows that the greatest equilibria in $E^{\ell}(\bar{z}^2)$ and $E^{\ell}(\bar{z}^1)$, respectively, can be found as the limit of the decreasing sequences $\{\bar{\pi}^{2,n}, \bar{w}^{\ell,2,n}\}$ and $\{\bar{\pi}^{1,n}, \bar{w}^{\ell,1,n}\}$ defined from $\bar{\pi}^{2,n+1} = \alpha(\bar{w}^{\ell,2,n}, \bar{z}^2)$, $\bar{w}^{\ell,2,n+1} = \alpha^{\ell}_{\vee}(\bar{\pi}^{2,n}, \bar{z}^2)$ and $\bar{\pi}^{1,n+1} = \alpha_{\vee}(\bar{w}^{\ell,1,n}, \bar{z}^1)$, $\bar{w}^{\ell,1,n+1} = \alpha^{\ell}_{\vee}(\bar{\pi}^{1,n}, \bar{z}^1)$, with $\bar{\pi}^{2,0} = \bar{\pi}^{1,0} = \bar{\pi}^0$ and $\bar{w}^{\ell,2,0} = \bar{w}^{\ell,1,0} = \bar{w}^{\ell,0}$ equal to the greatest elements, respectively, in $\mathcal{F}^{\infty}$ and $W^{\ell,\infty}$.

By Propositions A.13 and A.14, respectively, $\bar{\pi}^{2,1} = \alpha_\vee(\bar{w}^{\ell,0}, \bar{z}^2) \geq \alpha_\vee(\bar{w}^{\ell,0}, \bar{z}^1) = \bar{\pi}^{1,1}$ and $\bar{w}^{\ell,2,1} = \alpha^\ell_\vee(\bar{\pi}^0, \bar{z}^2) \geq \alpha^\ell_\vee(\bar{\pi}^0, \bar{z}^1) = \bar{w}^{\ell,1,1}$. Thus Proposition A.10 implies $\bar{w}^{\ell,2,2} = \alpha^\ell_\vee(\bar{\pi}^{2,1}, \bar{z}^2) \geq \alpha^\ell_\vee(\bar{\pi}^{1,1}, \bar{z}^2) \geq \alpha^\ell_\vee(\bar{\pi}^{1,1}, \bar{z}^1) = \bar{w}^{\ell,2,1}$. Because all $\bar{w}^{\ell,2,n}$ and $\bar{w}^{\ell,1,n}$ are increasing with respect to $\bar{\theta}$ for all n (see, again, the proof of Theorem A.1 as well as Proposition A.6), Proposition A.9 implies $\bar{\pi}^{2,2} = \alpha_\vee(\bar{w}^{\ell,2,1}, \bar{z}^2) \geq \alpha_\vee(\bar{w}^{\ell,2,1}, \bar{z}^2) \geq \alpha_\vee(\bar{w}^{\ell,1,1}, \bar{z}^1) = \bar{\pi}^{1,1}$. By induction, $(\bar{\pi}^{2,n}, \bar{w}^{\ell,2,n}) \geq (\bar{\pi}^{1,n}, \bar{w}^{\ell,1,n})$ for all n.

□

A.13 Empirical Predictions

We are interested in the share of individuals in the social group who decide to undertake a given course of behavior when their environment changes. Let $\bar{\pi}$ be an element of $\alpha(\bar{w}^\ell, \bar{z})$, i.e., a consistent sequence of aggregate mappings for the social group. The map $\phi : \Theta^t \times \mathcal{F}^\infty \times W^{\ell,\infty} \times Z^\infty \to [0,1]$, defined from

$$\phi_t(\theta^t, \bar{\pi}, \bar{w}^\ell, \bar{z}) = \int_X \hat{g}_t(x, \theta^t, \bar{\pi}, \bar{w}^\ell, \bar{z})[\pi_t(\theta^{t-1})](\mathrm{d}x), \text{ for } \theta^t \in \Theta^t, \quad t = 1, 2, \ldots, \tag{29}$$

gives the share of individuals in the social group who choose action $y = 1$ at time t, as a function of the partial history of leader's types observed up to time t, θ^t, the aggregate behavior of the social group, the behavior of the leader, and the expected sequence of exogenous circumstances $\bar{z}$.

A.13.1 The Role of the Leader's Action

We characterize how ϕ changes in response to the leader's action observed at time t.

Lemma A.9. *Suppose that Assumptions A.1-A.4 hold. Moreover, suppose that $\bar{w}^\ell$ and $\bar{z}$ are increasing in $\bar{\theta}$. If the payoff function exhibits increasing differences in (y, θ) and (x, θ), and $\bar{\pi}$ is increasing in $\bar{\theta}$, then $\bar{\phi}$ is increasing in $\bar{\theta}$. If the payoff function exhibits decreasing differences in (y, θ) and (x, θ), and $\bar{\pi}$ is decreasing in $\bar{\theta}$, then $\bar{\phi}$ is decreasing in $\bar{\theta}$.*

Proof. Fix t. Because $\bar{\pi}, \bar{w}^\ell$ and $\bar{z}$ are held fixed throughout the proof, we temporarily drop them from the notation. First, suppose that the payoff function exhibits increasing differences in (y, θ) and (x, θ), and $\bar{\pi}, \bar{w}^\ell$, and $\bar{z}$ are increasing in $\bar{\theta}$. Then Proposition A.5 and Corollary A.4 imply that $\hat{g}_t$ is increasing with respect to $\theta^t \in \Theta^t$ for all $t > 0$. Therefore for any $\theta^2_t, \theta^1_t \in \Theta^t$ such that

$\theta_t^2 \geq \theta^1$ and $t > 0$, we have

$$\begin{aligned}
\phi_t(\theta^{2,t}) &= \int_X \hat{g}_t(x, \theta^{2,t})[\pi_t(\theta^{2,t-1})](\mathrm{d}x) \\
&\geq \int_X \hat{g}_t(x, \theta^{1,t})[\pi_t(\theta^{2,t-1})](\mathrm{d}x) \\
&\geq \int_X \hat{g}_t(x, \theta^{1,t})[\pi_t(\theta^{1,t-1})](\mathrm{d}x) \\
&= \phi_t(\theta^{1,t}),
\end{aligned}$$

Second, suppose that the payoff function exhibits decreasing differences in (y, θ) and (x, θ), $\bar{\pi}$ is decreasing in $\bar{\theta}$, and $\bar{w}^\ell$ and $\bar{z}$ are increasing in $\bar{\theta}$. Then Proposition A.5 and Corollary A.4 imply that $\hat{g}_t$ is decreasing with respect to $\theta^t \in \Theta^t$ for all $t > 0$. Therefore for any $\theta_t^2, \theta_t^1 \in \Theta^t$ such that $\theta_t^2 \geq \theta^1$ and $t > 0$, we have

$$\begin{aligned}
\phi_t(\theta^{2,t}) &= \int_X \hat{g}_t(x, \theta^{2,t})[\pi_t(\theta^{2,t-1})](\mathrm{d}x) \\
&\leq \int_X \hat{g}_t(x, \theta^{1,t})[\pi_t(\theta^{2,t-1})](\mathrm{d}x) \\
&\leq \int_X \hat{g}_t(x, \theta^{1,t})[\pi_t(\theta^{1,t-1})](\mathrm{d}x) \\
&= \phi_t(\theta^{1,t}),
\end{aligned}$$

The same inequalities hold in the special case where the elements of $\bar{z}$ are constant functions, i.e., $z_t \in \mathbb{R}$ for all $t > 0$ (see Proposition A.12). Because the choice of t was arbitrary, this holds for all $t > 0$. $\square$

Lemma A.10. *Suppose that Assumptions A.1-A.4 hold. Moreover, suppose that $\bar{w}^\ell$ and $\bar{z}$ are increasing in $\bar{\theta}$. If the payoff function exhibits increasing differences in (y, θ) and (x, θ), and $\bar{\pi}$ is increasing, then $\mathbb{E}_t[\phi_{t+\tau}]$ and $\mathbb{E}_t[\phi_{t+\tau}]$ are increasing in θ^t on Θ^t for all $t > 0$. If the payoff function exhibits decreasing differences in (y, θ) and (x, θ), and $\bar{\pi}$ is decreasing, then $\mathbb{E}_t[\phi_{t+\tau}]$ and $\mathbb{E}_t[\phi_{t+\tau}]$ are decreasing in θ^t on Θ^t for all $t > 0$.*

Proof. Because $\bar{\pi}, \bar{w}^\ell$ and $\bar{z}$ are held fixed throughout the proof, we temporarily drop them from the notation. Fix t and τ, and use equation (29) to write:

$$\mathbb{E}_t \phi_{t+\tau+1} = \int_\Theta \cdots \int_\Theta \phi_{t+\tau+1}(\theta^{t+\tau+1}) \hat{Q}_{t+\tau}^{w^\ell}(\theta^{t+\tau}; \mathrm{d}\theta_{t+\tau+1}) \ldots \hat{Q}_t^{w^\ell}(\theta^t; \mathrm{d}\theta_{t+1})$$

for $\tau \geq 0$ and each $\theta^t \in \Theta^t$. First, focus on the case with increasing differences. By Lemma A.9, ϕ_t is an increasing function of $\theta^t \in \Theta^t$. By Lemma A.5, $\hat{Q}^\ell_t$ is stochastically increasing. It follows that

$$\int_\Theta \phi_{t+\tau+1}(\theta^{t+\tau+1})\hat{Q}^\ell_{t+\tau}(\theta^{t+\tau}; \mathrm{d}\theta_{t+\tau+1})$$

is an increasing function of $\theta^{t+\tau}$ for all $\tau \geq 0$. As a result, starting from any $\tau \geq 0$, we can integrate backwards and apply the same argument to obtain the conclusion. Similarly, with decreasing differences ϕ_t is an decreasing function of θ^t and the integral term above is a decreasing function of $\theta^{t+\tau}$ for all $\tau \geq 0$. The same inequalities hold in the special case where the elements of $\bar{z}$ are constant functions, i.e., $z_t \in \mathbb{R}$ for all $t > 0$ (see Proposition A.12). Because the choice of t and τ was arbitrary, the result holds for all $t > 0$.

□

Proposition A.16. *Let $y^\ell_t \in \{0, 1\}$ denote the observed leader's action at time $t > 0$, and suppose that Assumptions A.1-A.4 hold. Moreover, suppose that $\bar{w}^\ell$ and $\bar{z}$ are increasing in $\bar{\theta}$. If the payoff function exhibits increasing differences in (y, θ) and (x, θ), and $\bar{\pi}$ is increasing, then*

$$\mathbb{E}[\phi_{\vee,t+\tau}(\theta^{t+\tau})|\theta^t, y^\ell_t = 1] \geq \mathbb{E}[\phi_{\vee,t+\tau}(\theta^{t+\tau})|\theta^t, y^\ell_t = 0],$$

for all $t = 1, 2 \dots$, $\tau > 0$, and all $\theta^t \in \Theta^t$, where the expectation is taken with respect to density of $\theta^{t+\tau}$ conditional on θ^t and y^ℓ_t. If the payoff function exhibits decreasing differences in (y, θ) and (x, θ), and $\bar{\pi}$ is increasing, then the opposite inequality holds.

The result follows immediately from Proposition A.16 and the fact that Q^ℓ is stochastically increasing.

A.13.2 The Role of z

We now compare two triplets $(\bar{\pi}^2, \bar{w}^{\ell,2}, \bar{z}^2)$ and $(\bar{\pi}^1, \bar{w}^{\ell,1}, \bar{z}^1)$, where $\bar{z}^1$ and $\bar{z}^2$ are two sequences in Z^∞. While our ultimate interest is two compare two selected equilibria associated with $\bar{z}^1$ and $\bar{z}^2$, we state the result in general terms.

Lemma A.11. *Suppose that Assumptions A.1-A.4 hold, and let $\bar{w}^{\ell,2} \geq \bar{w}^{\ell,1}$, $\bar{z}^2 \geq \bar{z}^1$. If the payoff function has increasing differences in (y, θ), (x, θ), (y, z), and (x, z), $\bar{\pi}^2 \geq \bar{\pi}$, $\bar{\pi}^2$ and $\bar{\pi}^1$ are increasing, then $\phi_t(\theta^t, \bar{\pi}^2, \bar{w}^{\ell,2}, \bar{z}^2) \geq \phi_t(\theta^t, \bar{\pi}^1, \bar{w}^{\ell,1}, \bar{z}^1)$ for all $t > 0$ and all $\bar{\theta} \in \Theta^\infty$. If the payoff* *function has decreasing differences in (y,θ), (x,θ), (y,z), and (x,z), $\bar{\pi}^2 \leq \bar{\pi}$, and $\bar{\pi}^2$ and $\bar{\pi}^1$ are decreasing, then $\phi_t(\theta^t, \bar{\pi}^2, \bar{w}^{\ell,2}, \bar{z}^2) \leq \phi_t(\theta^t, \bar{\pi}^1, \bar{w}^{\ell,1}, \bar{z}^1)$ for all $t > 0$ and all $\bar{\theta} \in \Theta^\infty$.*

Proof. We only prove the first part of the proposition. Write

$$\begin{aligned}
\int_X \hat{g}_t(x, \theta^t, \bar{\pi}^2, \bar{w}^{\ell,2}, \bar{z}^2)\pi_t^2(\theta^{t-1})(\mathrm{d}x) &\geq \int_X \hat{g}_t(x, \theta^t, \bar{\pi}^1, \bar{w}^{\ell,2}, \bar{z}^2)\pi_t^2(\theta^{t-1})(\mathrm{d}x) \\
&\geq \int_X \hat{g}_t(x, \theta^t, \bar{\pi}^1, \bar{w}^{\ell,2}, \bar{z}^2)\pi_t^1(\theta^{t-1})(\mathrm{d}x) \\
&\geq \int_X \hat{g}_t(x, \theta^t, \bar{\pi}^1, \bar{w}^{\ell,1}, \bar{z}^2)\pi_t^1(\theta^{t-1})(\mathrm{d}x) \\
&\geq \int_X \hat{g}_t(x, \theta^t, \bar{\pi}^1, \bar{w}^{\ell,1}, \bar{z}^1)\pi_t^1(\theta^{t-1})(\mathrm{d}x)
\end{aligned}$$

for all $\theta^t \in \Theta^t$ and all $t > 0$, where $\hat{g}_t$ was defined in in (19). The first inequality follows because, by Corollary A.2, $\hat{g}_t$ is increasing in $\bar{\pi}$. Corollary A.2 also implies the second inequality because $\hat{g}_t$ is increasing in x and $\bar{\pi}^2 \geq \bar{\pi}^1$ in the stochastic dominance order. The third inequality follows from the fact that $\bar{\pi}^1$ is increasing: then Corollary A.5 implies that $\hat{g}_t$ is increasing in $\bar{w}^\ell$. The fourth inequality follows because, by Corollary A.6, $\hat{g}_t$ is increasing in $\bar{z}$.

□

The previous result considers each sequence of leader's types $\bar{\theta} \in \Theta^\infty$ separately, accounting for the fact that the leader's plan changes in response to variation in $\bar{z}$. However, it does not account for the fact that the leader's change of plan affects the likelihood of observing each of those sequences in the future.

Proposition A.17. *Suppose that Assumptions A.1-A.4 hold, and let $\bar{w}^{\ell,2} \geq \bar{w}^{\ell,1}$, $\bar{z}^2 \geq \bar{z}^1$. Moreover, suppose that $\bar{w}^{\ell,2}, \bar{w}^{\ell,1}, \bar{z}^2, \bar{z}^1$ are increasing in $\bar{\theta}$. If the payoff function has increasing differences in (y,θ), (x,θ), (y,z), and (x,z), $\bar{\pi}^2 \geq \bar{\pi}$, $\bar{\pi}^2$ and $\bar{\pi}^1$ are increasing, then*

$$\mathbb{E}[\phi_{t+\tau}(\theta^{t+\tau}, \bar{\pi}^2, \bar{w}^{\ell,2}, \bar{z}^2)|\theta^t] \geq \mathbb{E}[\phi_{t+\tau}(\theta^{t+\tau}, \bar{\pi}^1, \bar{w}^{\ell,1}, \bar{z}^1)|\theta^t],$$

for all $t = 1, 2\dots$, $\tau > 0$, and all $\theta^t \in \Theta^t$, where the expectations are taken with respect to densities of $\theta^{t+\tau}$ conditional on θ^t. If the payoff function has increasing differences in (y,θ), (x,θ), (y,z), and (x,z), $\bar{\pi}^2 \geq \bar{\pi}$, $\bar{\pi}^2$ and $\bar{\pi}^1$ are increasing, the the opposite inequality holds.

Proof. Use equation (29) to write:

$$\mathbb{E}_t\phi_{t+\tau+1} = \int_\Theta \cdots \int_\Theta \phi_{t+\tau+1}(\theta^{t+\tau+1}, \bar{\pi}, \bar{w}^\ell, \bar{z})\hat{Q}^{w^\ell}_{t+\tau}(\theta^{t+\tau}; \mathrm{d}\theta_{t+\tau+1}) \ldots \hat{Q}^{w^\ell}_t(\theta^t; \mathrm{d}\theta_{t+1})$$

for $\tau \geq 0$ and each $\theta^t \in \Theta^t$, where $\{\hat{Q}^{w^\ell}_t\}_{t>0}$ is also evaluated at $\bar{w}^\ell_\vee$.

We only prove the first part of the proposition. First, Lemma A.11 shows that $\phi_t(\theta^t, \bar{\pi}^2, \bar{w}^{\ell,2}, \bar{z}^2) \geq \phi_t(\theta^t, \bar{\pi}^1, \bar{w}^{\ell,1}, \bar{z}^1)$. Second, because $\bar{w}^{\ell,2}$, $\bar{w}^{\ell,1}$, $\bar{z}^2, \bar{z}^1$ are increasing with respect to $\bar{\theta}$. Lemma A.9 shows that both are increasing functions of θ^t on Θ^t for all $t > 0$. Third, Lemma A.5 shows that $\hat{Q}^{w^{\ell,2}}_t$ and $\hat{Q}^{w^{\ell,2}}_t$ are stochastically increasing in $\theta_t \in \Theta$ for all $t > 0$. Fourth, because $\bar{w}^{\ell,2} \geq \bar{w}^{\ell,1}$, Lemma A.4 implies that $\hat{Q}^{w^{\ell,2}}_t \geq \hat{Q}^{w^{\ell,1}}_t$ (or, alternatively, $\bar{q}^2 \geq \bar{q}^1$). It follows that

$$\int_\Theta \phi^2_{\vee,t+\tau+1}(\theta^{t+\tau+1}, \bar{z}^2)\hat{Q}^{w^{\ell,2}}_{t+\tau}(\theta^{t+\tau}; \mathrm{d}\theta_{t+\tau+1}) \geq \int_\Theta \phi^1_{\vee,t+\tau+1}(\theta^{t+\tau+1}, \bar{z}^1)\hat{Q}^{w^{\ell,1}}_{t+\tau}(\theta^{t+\tau}; \mathrm{d}\theta_{t+\tau+1})$$

Furthermore, A.9 says that both sides of this inequality are increasing functions of $\theta^{t+\tau}$ for all $t = 1, 2\ldots$ and $\tau \geq 0$. As a result, starting from any $\tau \geq 0$, and keeping $\hat{Q}^{w^{\ell,2}}_{t+\tau}$ and $\hat{Q}^{w^{\ell,1}}_{t+\tau}$ fixed, we can integrate backwards and apply the same argument to obtain the conclusion. □

B Theory Appendix: Proofs of the Results in the Main Text

Proof of Proposition 1. The proof is an application of Proposition A.16 to the case where the payoff function has increasing differences, with $\bar{z} = \{z_t\}$ such that each z_t is a constant function over Θ^t and $(\bar{\pi}, \bar{w}^\ell)$ equal to the greatest discrimination equilibrium with a leader $(\bar{\pi}_\vee, \bar{w}^\ell_\vee)$ in $E^\ell(\bar{z})$. Proposition A.11 shows that $(\bar{\pi}_\vee, \bar{w}^\ell_\vee)$ are increasing with respect to $\bar{\theta}$, so that all the hypotheses of the Proposition are satisfied. □

Proof of Proposition 2. The proof is an application of Proposition A.16 to the greatest equilibrium in $\alpha(\bar{w}^\ell, \bar{z})$, with $(\bar{w}^\ell, \bar{z})$ equal to the greatest discrimination equilibrium with a leader $(\bar{w}^\ell_\vee, \bar{\pi}_\vee)$ in $E^\ell(\bar{z})$. Proposition A.11 shows that $(\bar{w}^\ell_\vee, \bar{\pi}_\vee)$ are increasing with respect to $\bar{\theta}$. Then, Proposition A.7 implies the monotonicity properties of the greatest element in α. □

Proof of Proposition 3. The proof is an application of Proposition A.15 to the case where the payoff function has increasing differences, with $\bar{z}^1 = \{z^1_t\}$ and $\bar{z}^2 = \{z^2_t\}$ such that each z^1_t and z^2_t is a constant function over Θ^t and $(\bar{\pi}^1, \bar{w}^{\ell,1})$ and $(\bar{\pi}^2, \bar{w}^{\ell,2})$ are equal to the greatest discrimination equilibrium with a leader in $E^\ell(\bar{z}^1)$ and $E^\ell(\bar{z}^2)$, respectively. □

Proof of Proposition 4. The proof is an application of Proposition A.15 to the greatest equilibria in $\alpha(\bar{w}^{\ell,1}, \bar{z}^1)$ and $\alpha(\bar{w}^{\ell,2}, \bar{z}^2)$, with $(\bar{w}^{\ell,1}, \bar{z}^1)$ and $(\bar{w}^{\ell,2}, \bar{z})^2$ equal to the greatest discrimination equilibria with a leader in $E^\ell(\bar{z}^1)$ and $E^\ell(\bar{z}^2)$. □

C Theory Appendix: Intermediate Results

Lemma C.1. *Let the integrable function $f : X \times Z \to \mathbb{R}$ exhibit increasing differences, and let the transition kernel $Q : Y \times \mathcal{B}(X) \to [0, 1]$ be stochastically increasing. Then:*

$$\int_X f(x', z)Q(x, y; \mathrm{d}x')$$

exhibits increasing differences in (y, z) at each $x \in X$.

Proof. Pick $z_1, z_2 \in Z$ such that $z_2 \geq z_1$. Because $f(x, z)$ exhibits increasing differences,

$$f(x_2, z_2) - f(x_2, z_1) \geq f(x_1, z_2) - f(x_1, z_1)$$

for all $x_1, x_2 \in X$ such that $x_2 \geq x_1$. Therefore the function $f(\cdot, z_2) - f(\cdot, z_1)$ is increasing on X.

Because Q is stochastically increasing, we have

$$\int_X \left[f(', z_2) - f(x', z_1)\right] Q(y_2; \mathrm{d}x') \geq \int_X \left[f(x', z_2) - f(x', z_1)\right] Q(y_1; \mathrm{d}x')$$

for all $y_1, y_2 \in Y$ such that $y_2 \geq y_1$. Rearranging terms:

$$\begin{aligned} &\int_X f(x', z_2)Q(y_2; \mathrm{d}x') - \int_X f(x', z_1)Q(y_2; \mathrm{d}x') \\ \geq &\int_X f(x', z_2)Q(y_1; \mathrm{d}x') - \int_X f(x', z_1)Q(y_1; \mathrm{d}x') \end{aligned}$$

□

The following Lemma summarizes an argument in Bergin and Bernhardt (1992).

Lemma C.2. *The set $\mathcal{F}^\infty$ is compact.*

Proof. The set $\mathcal{P}$ is compact in the topology of weak-convergence since X is compact (Prokhorov's theorem). The set $\mathcal{F}^t = \times_{\theta^t \in \Theta^t} \mathcal{P}$, and the set $\mathcal{F}^\infty = \times_{t=1}^\infty \mathcal{F}^t$ are compact in the corresponding product topologies (Tychonoff's theorem). □

Kertz and Rösler (2000) prove a completeness property of $\mathcal{P}$ with respect to all its bounded subsets. Because when X is bounded all subsets of $\mathcal{P}$ are bounded, we have:

Lemma C.3. *The set of probability measures $\mathcal{P}$ on X is a complete lattice.*

Proof. See Lemma 3.1, Kertz and Rösler (2000). □

Lemma C.4. *The set $\mathcal{F}^\infty$ is a complete lattice*

Proof. The set $\mathcal{P}$ is a complete lattice (Lemma C.3). Since $\pi_{t+1}(\theta^t) \in \mathcal{P}$ for all $\theta^t \in \Theta^t$, π_{t+1} is an element of $\times_{\theta^t \in \Theta^t}\mathcal{P}$. The direct product of complete lattices is a complete lattice in the pointwise order. Hence $\mathcal{F}^t$ and $\mathcal{F}^\infty$ are complete lattices in the pointwise order. □

The next Lemma is presented in Light and Weintraub (2022) and is itself an application of more general results in Serfozo (1982).

Lemma C.5 (Light and Weintraub (2022))**.** *Assume that $F_n : X \to \mathbb{R}$ is a uniformly bounded sequence of functions. If F_n converges continuously to F and p_n converges weakly to p, then:*

$$\lim_{n\to\infty} \int_X F_n(x)[p_t^n(\theta^{t-1})](\mathrm{d}x) = \int_X F(x)[p_{t+1}^\infty(\theta^t)](\mathrm{d}x)$$

Pick an f that is continuous and bounded. Then F_n is uniformly bounded for all $n = 1, 2 \ldots$ by $\sup_{x\in X} f(x)$. Also, F_n converges continuously to F because Q has the Feller property and because $\hat{g}_t$ is jointly continuous. Since the argument holds for an arbitrary choice of f it must hold for all f.

D Data Appendix

D.1 Trump's Tweets Using the Expression "Chinese Virus"

Figure D1: Trump's Tweets using the expression "Chinese Virus"

D.2 Details on Data Construction

D.2.1 Construction of the Main Samples of White and Chinese Users

To construct the dataset we perform four steps: (i) retrieve lists of users for the White and Chinese group based on Twitter bios, (ii) retain only influential users, (iii) retain only users geocoded in the U.S., (iv) detect and drop noisy profiles from each group. We discuss each step in more details below.

Retrieving lists of users for the White and Chinese group based on Twitter bios. To retrive lists of users belonging to the White and Chinese groups we used the platform https://followerwonk.com/, that is a website providing tools and analytics to manage Twitter accounts. In particular, the website allows searching Twitter accounts based on specific keywords appearing in the (self-reported) user bio, returning up to $50,000$ accounts for each search, sorting the accounts according to their number of followers. We therefore searched the Twitter bios based on keywords that signal the belonging to our two groups of interest. Specifically, for the White group we used the following search patterns (with number of retrived profiles in parentheses): "american" (50,000), "white american" (466), "christian american" (742), "proud american" (7,375), "american patriot" (3,091). To retrieve users of the Chinese group we searched for the keyword "chinese" and find 18,376 profiles.

The white american group in the paper comprises all accounts in the *white_american*, *proud_american*, *christian_ american*, *american_patriot* lists, plus all accounts in the *american* list whose description also contains either "white", "proud", "christian" or "patriot". Since our data series starts on January 6, 2020 as a preliminary filter we drop all accounts with no tweets created after January 1 (this information is provided directly by followerwonk).

Focus on influential users. To exclude individuals with very low level of Twitter activity or users that wrongly attributed to either groups, the searches in the previous step were carried out restricting results, respectively, to accounts with more than 100 followers and to accounts whose descriptions *do not* contain the following expressions: "porn", "18+", "restaurant", "food". Once the lists are retrieved, we additonally restrict our attention to *influential* users, that is either users ranked in the top 2,500 for number of followers for their specific group, *or* reporting in their bio selected keywords for arts, sports, culture, media, politics, etc.. The list of keywords follows.

> feminist, exiled, dissident, human rights, activist, political, commentator, non-profit organization, Chinese culture, FreeSpeech, humanrights, Observer, Thinker, Society, speaker, democracy, Women's empowerment, Philanthropist, politics, victims, Community, civil rights, reporter, editor, communication, communist, peace, imperialism, movement, justice, humanitarian, liberty, free market, entrepreneur, immigrant, civic, racism, racist, refugee, oppression, freedom, opinion, climate, Yang, president, conservative, campaining, conservative, thinker, human rights, democratic values, justice, fair treatment, trump, republican, democratic, liberal, leftist, editor, columnist, author, feminist, pro life, political, activist, community, tea party, community, freedom, lefty, peace, Libertarian, progressive, lgbt, democracy, better world, lesbian, gay, patriot, politics, socialist, nationalist, secular, identity, Social Entrepreneur, proud, proudly, so-

cial justice, security, sustainable world, freedom, religious network, civil rights, action, proud, Thinker, ghetto, immigrants, empowerment, empower, empowering, liberty, pastor, christian, activist, voice, women, church, fighter, racism, injustice, non-profit organization, resistance , militant, Homophobia, community, LGBT centre, political issues, inspiration, activist, GayRights, politics , feminism , human rights, proud, opinionated, police abuse, leftist, democrat, activist, journalist, lesbian, non-government organization, pacifist, commentator, columnist, movement, immigrant, freedom, fighter, human rights, women, empower, thinker, writer, voice, speaker, politician, social worker, blogger, patriotic, poet, independent , poet, columnist, writer, resistance , news, human rights, independent, singer, journalist, tolerance, revolution , activism, activist, movement, feminist, voice, democratic, fighter, women, civil rights, advocacy, political issues, advocate, patriotic, nationalist, influencer, community, independent, proud, empower, advance, impacting, leader, blog, blogger, dreamer, fighter, radio, influencer, proud, news, politics, opinions, rights, writer, trump, equality, actor, player, champion, medalist, empowering, feminist, writer, activist, journalist, free press, traveler, thinker, author, democrat, opinions, issues, trending topics, youtuber, movement, actor, actress, entrepreneur, player, journalist, empowerment, blogger, cosplayer, singer, animator, illustrator, writer, politics, radio, issues, patriot, game developer, campaign, equality, HumanRights, feminist, community, politics, rights, progressive, civil, Pride, activist, advocate, freedom, marginalized, anti-racism, inclusion, LGBT people, journalist, justice, human rights, bloger, activist, immigrant, artist, Trump, conservative, refugee, MAGA, proud, descent, empower, constitutionalist, prisoner, legal, advocate, leader, freedom, issue, KAG, Actress, Artist, Actor, Director, Author, MasterChef, Woman, Blogger, Influencer, Winner, Model, Pediatrician, Meme, Journalist, ICU, Creative, Patriot, Constitution, American, MAGA, femminist, immigrant, governor, pensador, diplomat, activist, Christian, Reporter, Supporter, Envirorment, Energy, director, artista, MAGA, human rights, civil rights, immigrant, lawyer, feminist, advocacy, immigration, leader, cultural, politics, impact, civic engagement, activist, freedom, inclusion, advocate, influencer, blogger, promote, actress, feminist, immigrant, progressive, activist, content creator, freedom, community, leader, empower, protest, social justice, activist, actress, music, journalist, conservative, artist, blogger, columnist, writer, opinions, progressive, feminist, activist, politics, resistance , artist, activist, empowerment, empower, blogger, obama, Trump, womanhood, racism, democrat, justice, independent, voice, nationalist, writer, movement, Journalist, humanitarian, feminist, black lives matter, activist, liberal, blogger, obama, peace, open minded, voice.

We include a user in the sample provided that she meets either of the following two conditions:

- the self-reported description contains at least one of the keywords in the final list;
- the user is in the top 2,500 users in her group in terms of number of followers;

This is the sample of users for whom we have downloaded the timelines. However, we impose further restrictions on the sample of users who enter our main analysis.

Geolocation of users. We include in the sample only users who are located in the US. The methods used to determine the user's location differ across groups: White Americans are assumed to be located in the US while for Chinese users we proceeded in two steps. First, we hand-coded the user's country based on the self-reported location. This variable takes values "USA" or "Other", whenever the country could be unambigously deduced from the self-reported location, and missing otherwise. Second, for all users with missing country of residence after the first step, we infer their location from the location of their friends (Barberá et al., 2019). In particular, we consider as American users those that: (i) have a share of friends with missing location below 60%; (ii) have at least 10 friends geolocated in the US; and (iii) have a share of US friends greater than the share of friends from elsewhere.

Excluding noisy profiles. We exploit a second set of keywords to detect and drop noisy profiles from the sample. These profiles are returned in the followerwonk lists (and thus contain one of the group-specific search patterns), make it through the filters used to detect influential users, and still are very unlikely to be members of the corresponding group. While most of these "excluding" keywords are group-specific, some of them hold for all groups. The corresponding regex patterns are: "gay content", "porno", "(adults | gay) only", "adult entertainment", "+21".

D.2.2 Keywords Used to Indentify Relevant Subsets of Users

To rule out that main results of the paper are driven by particular sets of users, we defined for each group a subsample of users that are particularly involved into politics or activism (of any type), based on the presence of the following keywords:

political, democracy, politics, communist, yang, president, conservative, campaining, democratic values, trump, republican, democratic, liberal, leftist, tea party, lefty, libertarian, progressive, socialist, nationalist, proud, proudly, social justice, patriot, political issues, politics, democrat, politician, patriotic, leader, campaign, pride, maga, constitutionalist, kag, constitution, governor, diplomat, supporter, obama, feminist, exiled, dissident, human rights, activist, non-profit organization, freespeech, humanrights, society, women's empowerment, victims, philanthropist, community, civil rights, peace, imperialism, movement, justice, humanitarian, liberty, civic, racism, racist, immigrant, refugee, oppression, freedom, climate, fair treatment, pro life, lgbt, better world, lesbian, gay, social justice, sustainable wolrd, action, ghetto, immigrants, empowerment, empower, empowering, voice, women, fight, injustice, resistance, militant, homophobia, lgbt centre, political issues, ispiration, gayrights, feminism, non-government organization, pacifist, social worker, independent, tolerance, revolution, activism, advocacy, advocate, advance, impacting, rights, equality, free press, issues, civil, marginalized, anti-racism, inclusion, lgbt people, woman, femminist, supporter, environment, energy, civic engagement, promote, protest, womanhood, black lives matter, fighter, police abuse, influencer, blog, leader, dreamer, opinions, blogger, trending topics, youtuber, prisoner, legal issue, femminist, impact, community.

D.2.3 Keywords: Assimilation

As example of the assimilation content of tweets based on keywords, this subsection reports the text of some tweets including the keywords "Chinese American" or "Asian American" from a random sample of all the tweets including either keywords:

- "We Asian Americans are not the virus, but we can be part of the cure, @AndrewYang writes"
- "Every year on Dr. King's Anniversary I go into meditation. What role should we play as an ethnic minority Asian American to remember his greatness? In addition to encouraging the community to vote, it is love and tolerance. God bless America."
- "An Open Letter to the Chinese American Community from Four Recent Johns Hopkins Graduates Who Call China and America Home"
- "I wish #AndrewYang could read the over 1,000 accounts of hate Asian Americans have endured just trying to survive like everyone else. We don't need to prove our worth or we belong @CAAsanfrancisco - The Washington Post"

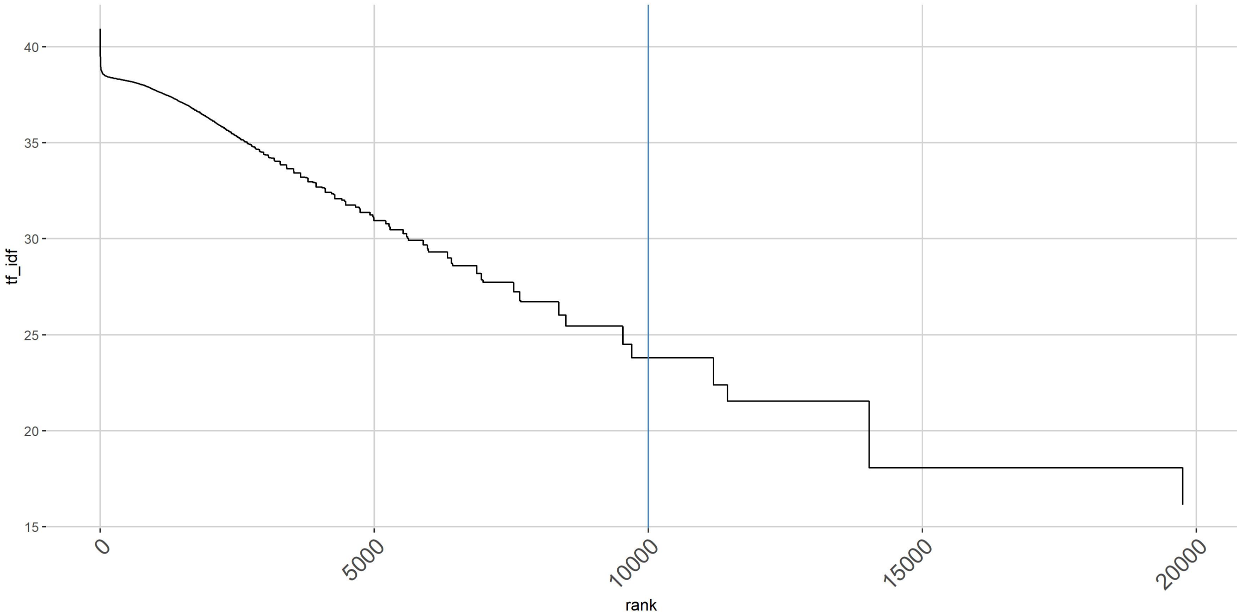


Figure D2: Tf-Idf in the training dataset (Founta et al., 2018)

- "@jennyyangtv @AndrewYang BTW, our local Chinese community is doing a lot for our greater community here. Our leaders have organized donations, securing masks and other protective fears etc delivered to our local hospitals. Not because we are Chinese Americans, but because we want to do our best."
- "we hope that in all our stories listeners understand one of the major points of our show is that asian americans are already an inherent part of the history and culture of this country and do not ever need to prove their worthiness to belong"

D.3 Supervised Machine Learning: Discrimination

Figure D2 shows the tf-idf index for the training dataset with hateful tweets (left panel) and for the training dataset with abusive tweets (right panel). Based on visual inspection, we keep the top 8500 words in the former dataset, and the top 12000 words in the latter dataset (as shown with the blue line in the figure). The next table shows the confusion matrix.

D.4 Supervised Machine Learning: Assimilation

This subsection reports additional details on the Word2Vec classification exercise. First, Table D2 reports the set of assimilation sentences that we used as baseline text to classify the assimilation

Table D1: Confusion Matrix

		Prediction	
		normal	other
Reference	normal	$15,377$	598
	other	$2,375$	$6,755$

content of the tweets, together with their original source. In particular, we drew on the following list of journal articles and websites:

- Kibria, Nazli (2000) "Race, ethnic options, and ethnic binds: Identity negotiations of second-generation Chinese and Korean Americans", *Sociological perspectives* 43 (1): 77-95.
- Lee, Stacey J. (1994) "Behind the model-minority stereotype: Voices of high-and low-achieving Asian American students", *Anthropology & Education Quarterly* 25 (4): 413-429.
- Liu, Shuang (2015) "Searching for a sense of place: Identity negotiation of Chinese immigrants", *International Journal of Intercultural Relations* 46: 26-35.
- Ng, T. K., et al. (2014) "A transnational bicultural place model of cultural selves and psychological citizenship: The case of Chinese immigrants in Britain", *Journal of Environmental Psychology* 40: 440-450.
- [BA - David Ho] Webpage Becoming American: The Chinese Experience. David Ho Transcript.
- [BA - Gish Jen] Becoming American: The Chinese Experience. Gish Jen Transcript.
- [BA - Helen Zia] Webpage Becoming American: The Chinese Experience. Helen Zia Transcript.
- [BA - Maya Lin] Webpage Becoming American: The Chinese Experience. Maya Lin Transcript.
- [BA - Sam Ting] Webpage Becoming American: The Chinese Experience. Sam Ting Transcript.
- [BA - Shirley Young] Webpage Becoming American: The Chinese Experience. Shirley Young Transcript.

- [OWE] Webpage Di, Julia (2012) “Chinese-American Identity”.
- [H - Sam Louie] Webpage Robbins, Jefferson (2015) “On Being Asian American: An Interview with Sam Louie”.
- [PH] Webpage Tse, Melissa. “The Twists and Turns of the Chinese-American Identity”.
- [CNN] Webpage Kallingal, Mallika (2009) “These Asian Americans faced racism growing up, but they won’t let it define them”.

Second, to check if the algorithm and definition of the variable correctly identify assimilation content, we took a closer look to 100 tweets randomly chosen between the tweets classified as assimilation tweets based on the algorithm and classification rules (see text for more details). In particular, 56% of the tweets could be manually classified as reporting assimilation content as they were tweeting about US-specific topics including politics and specific issues of the American society (50%), or tweeting about against the CCP (32%). Below some examples of tweets with assimilation content, explicitly expressing a feelings of “Americanness”:

- “@LUCYDZERO1 Thanks. So don’t rush, everyone can learn English well enough to live well in the US if they have patience and work hard to learn. Especially in the US there are many very convenient free friendly schools and environments to learn English.”
- “After enduring decades of exclusion, racism and discrimination that include some of the darkest chapters of American history, AsianAmericans entered 2020 with reason for optimism on the political front... and then along came the Coronavirus.”
- “@teasoonreal @ShumPris @WSJ Interesting, I’ve been here for so long. Earn my respect by hard works and, is there asshole? I guess so, everywhere has them. Is discrimination widespread? Not even close. Actually from what I’m looking at, I’m easier to get discriminated in China compare to US, at this time.”

Table D2: Assimilation sentences

Source	text
Kibria (2000)	I think it's really important for people to realize that we're not foreigners.
Kibria (2000)	As far as my nationality, I'm a U.S. citizen. As far as my ethnicity, I'm a [Chinese] American.
Kibria (2000)	I'm not usually very conscious of being Chinese. When I am conscious it's because I've been reminded of it.
Kibria (2000)	I don't think I have any kind of Chinese accent.
Kibria (2000)	I wanted to be an American like everyone else. And the thing of it is that I was actually completely American.
Kibria (2000)	I dressed cool, I learned how to dance, I became the class clown. There were a few Japanese and Chinese in my school, and I looked really carefully to see how they would act, and then I would do exactly the opposite! And it worked! I was one of the most popular kids in school.
Kibria (2000)	I realized then that I was really good at projecting different identities. I could copy hand gestures, accent, you name it, down to the smallest detail.
Kibria (2000)	People tell me that I'm the most Americanized Korean they've ever met.
Kibria (2000)	How do you feel about that? Hey, it doesn't bother me. I've never felt that I've been discriminated against for being Korean.
Kibria (2000)	I mean, I talk like an American; I don't have broken English. I act like everybody else.
Kibria (2000)	My husband [who is Chinese American] and I feel like it's important to give the kids strong Korean roots. But we also want them to have all the advantages, some of which we didn't have growing up in an immigrant family.
Kibria (2000)	So if you know all the little almost invisible things about American culture, you're likely to have an easier time. You're accepted better.
Lee (1994)	When I first came to [the] U.S., they said I should get-should hang out with American kids so I could get Americanized. So, I hang out with American
Lee (1994)	I have experiences that are similar to other Asians that live in America: that my culture is not all Asian and it's not all American. It's something entirely different.
Lee (1994)	When I say I'm Asian American, I feel like I establish a root for myself here.
Lee (1994)	Being Asian American is like a way to feel I belong.
Liu (2015)	I consider myself [American] Chinese I guess, because it's obvious that I look like Chinese
Liu (2015)	I consider myself an [American] because I am a citizen
Liu (2015)	In terms of identity I would give myself the [American] label because that's where I have spent most of my life.
Liu (2015)	Further, I feel most aligned with being an [American] in terms of citizenship and the State's recognition of me as a citizen
Liu (2015)	I consider myself ethnically Chinese but culturally [American]
Liu (2015)	There are some people that we call ABC [America born Chinese], even though their skin and their physical appearance are Chinese, I can totally tell they are definitely not Chinese at all. It s the way they talk and behave.
Liu (2015)	I consider myself half Chinese half [American].
Liu (2015)	I feel [American] when I'm with my friends since they all grew up in [America]
Liu (2015)	We are more Chinese in [America] but more [American] in China
Liu (2015)	My parents still regard me as Chinese, but I don't agree with them.
Liu (2015)	At work, I'm [American] because I don't want to be treated as a minority. I want to tell them I'm capable of doing what they are capable of doing
Liu (2015)	I think I have the ability to switch my mind towards [American] or Chinese.
Liu (2015)	I do things in an [American] way at work and with [American] friends.
Liu (2015)	I try not to show my [American] side when I'm in China: like with close [American] friends we sometimes humiliate or makefun of each other as some sort of entertainment but I never do that with Chinese friends because they could get very offended.
Liu (2015)	And I always split bills with [American] friends when doing things together, but Chinese friends in China would think stingy of me if I did so with them
Liu (2015)	I try to fit into the groups of people I'm meeting.
Liu (2015)	If there are more Chinese people I may act more Chinese, but if there are more [American] people, I'll be more [American]; just to fit in the group
Ng et al. (2014)	It is important for me to feel being a member of the community
Ng et al. (2014)	I feel part of a combined Chinese and [American] culture
Ng et al. (2014)	I feel I can move freely between the Chinese and [American] cultures without feeling any conflict
BA - David Ho	think, the fact that our kids are clearly American that we are American.
BA - David Ho	We are Americans; we're just of a different heritage.
BA - Gish Jen	And of course, as a community it was completely committed to being open and embracing and so on
BA - Gish Jen	We've shown that we are American in every way of our– we like hot dogs. We like baseball. We like fast cars. The issue still for us is being accepted as an American
BA - Helen Zia	For me, as a second-generation, American-born Chinese, that is the struggle of our parents' generation. They want us to hold on tight to those elements of Chinese culture.
BA - Helen Zia	But I'd never been there. I identified much more with hotdogs, apple pie, baseball, Chevrolet.
BA - Helen Zia	The relationship between child and Chinese parent was often a struggle between the two cultures.
BA - Helen Zia	The values I knew and was learning were as an American kid. I was also trying to bridge this notion of being Chinese.
BA - Helen Zia	For me, it was the opposite. America is my familiar world and China is something I have to learn.
BA - Maya Lin	I wanted to fit in; I wanted to be American,
BA - Maya Lin	And there could be a white German who's English is okay. He could have just traveled here yesterday and they will assume he's American.
BA - Sam Ting	It's my theory that one becomes an American at that moment which for good or ill, one challenges authority. Challenges tradition
BA - Sam Ting	if you no longer celebrate Chinese New Year and then you realize you have completely changed
BA - Shirley Young	I always have felt I owe a great deal to this country
OWE	They never tell you about the rejection from eachside: my Chinese cousins regard me as the American, while my American friends understand that I am Chinese.
H - Sam Louie	Growing up in America, you're taught to personalize everything, to individuate, to be your own person,
PH	I realized then that my heritage as a Chinese-American involuntarily tied me to a larger national identity.
CNN	The negative incidents of racial discrimination in my childhood are no longer something that I see as an integral or even influential part of my growing up.

D.5 Unsupervised Machine Learning Exercises

The LDA is a generative probabilistic model for documents. Each document is represented as a vector of topics shares, where each topic is a probability distribution over words. Thus, "generating a document", according to the LDA algorithm, involves the following steps: *1.* select the term distribution (β) for each topic from a Dirichlet distribution with hyperparameter δ; then, for each document: *2.* select the number of words (N) in the document (for instance, from a Poisson distribution); *3.* select the topic distribution (γ) from a Dirichlet distribution with parameter α; *4.* for each of the N words, select the topic assigned to that word using γ, obtained in step 3, and select a term using β, obtained in step 1. This process, repeated for all documents in the corpus, delivers the probability of observing a given corpus in the data, and maximum likelihood techniques can then be used to recover γ and β.

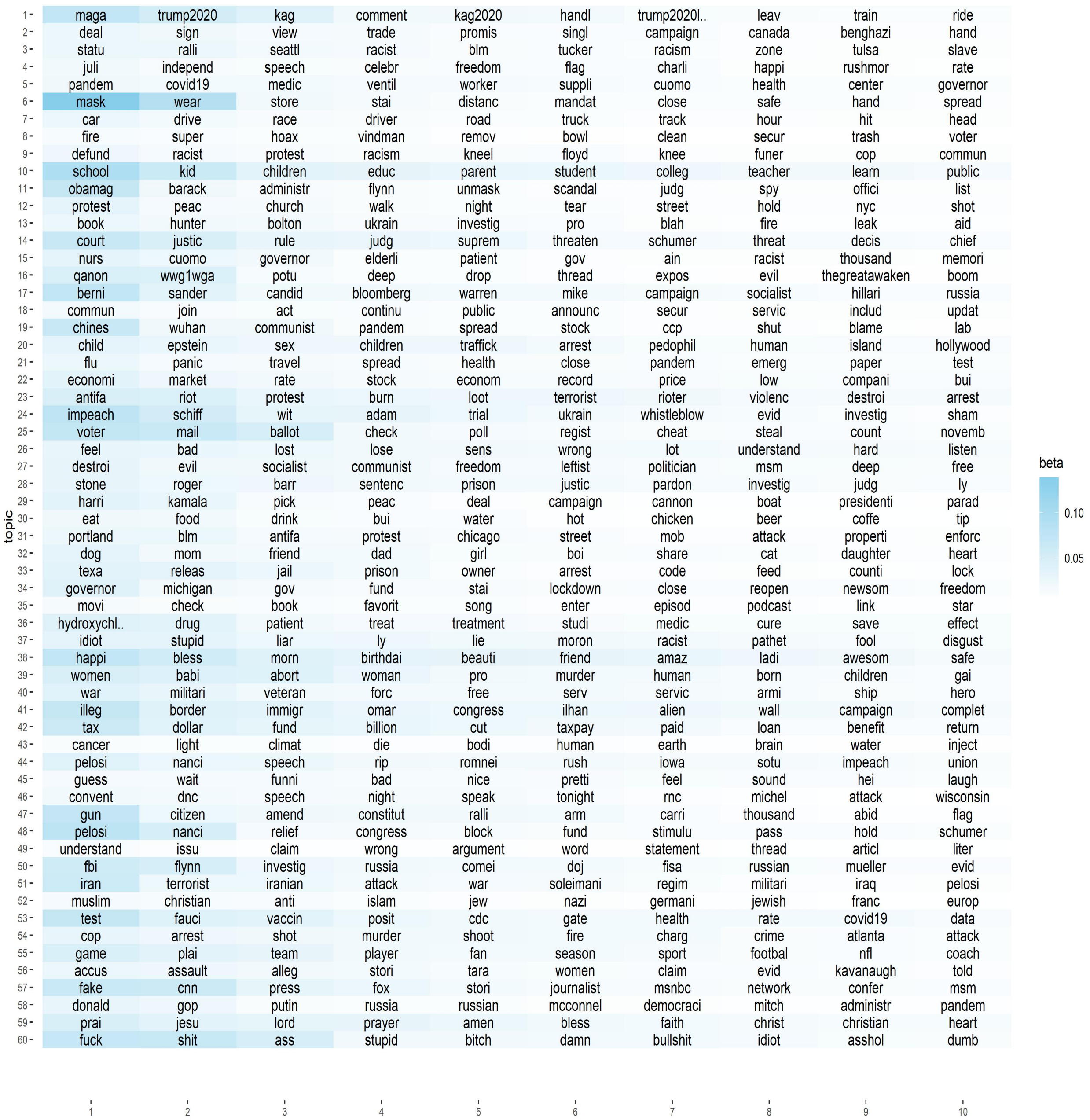
D.5.1 Discrimination

Figure D3 displays the list of topics with the related 10 keywords with highest weight. For the topic of interest we report below some examples of the text included in the 30 documents for which the weight of the topic was highest.

Example of tweeted text for topic 19 "Discrimination"

- "The lying and disturbing effort to protect China is astounding You are aiding abetting the enemy It is an undisputed fact that the Wuhan virus originated from Wuhan China This is a bioweapon intentionally or accidentally released"
- "I take a daily medication and every time I put it in my mouth I think China is slowly poisoning me Please bring our medication production back to our country I want to walk into an American pharmacy and buy a product I can trust I don t trust China
- "[...] Chinese virus Chinese virus Chinese virus Chinese virus [x16 ...] We are going to win and America will be stronger than ever before"
- "Many Americans are frightened by the #WuhanCoronavirus. [...] Trump beat Fake News to a pulp and he's kicking CHINA virus in the balls! [...] No one thinks Chinese Americans are to blame for the Wuhan virus. Holding Communist China accountable for their role in worsening this pandemic is critical."

Figure D3: Topics of the White group



D.5.2 Assimilation

Figure D4 displays the list of topics with the related 10 keywords with highest weight. For the topic of interest we report below some examples of the text included in the 30 documents for which the weight of the topic was highest.

Figure D4: Topics of the Chinese group

topic	1	2	3	4	5	6	7	8	9	10
1	got7	jinyoung	jackson	dye	song	stan	miss	album	notbythemoon	happi
2	hong	kong	law	nation	taiwan	secur	hongkong	beij	tiktok	govern
3	war	parti	polit	power	countri	system	leader	fight	govern	anti
4	vote	berni	biden	support	candid	sander	campaign	yang	win	warren
5	share	hope	care	feel	wear	mask	lot	read	blog	support
6	free	content	link	pic	custom	plai	daili	subscrib	check	thread
7	american	asian	japan	commun	japanes	racism	countri	support	anti	militari
8	trump	presid	democrat	biden	vote	american	america	elect	joe	republican
9	book	read	histori	studi	publish	research	write	review	project	paper
10	lot	watch	hope	match	gui	feel	start	pretti	fan	win
11	game	plai	team	player	sport	watch	miss	die	win	kobe
12	report	question	stori	public	journalist	issu	articl	read	statement	polit
13	ccp	communist	parti	ccpviru	truth	hsk	american	govern	pandem	america
14	home	citi	stai	close	week	hous	park	chicago	break	busi
15	fuck	women	tran	black	white	movi	liter	feel	hate	girl
16	famili	god	friend	happi	life	birthdai	kid	mother	mom	parent
17	fuck	shit	ass	episod	season	girl	boi	damn	bitch	frame
18	student	school	learn	class	onlin	word	languag	teacher	join	teach
19	book	artist	link	draw	drop	song	share	support	comic	listen
20	monei	job	lot	tax	fund	public	rent	care	system	insur
21	tool	code	found	bugbounti	vulner	secur	bug	hack	check	file
22	read	word	english	translat	languag	sound	form	sens	write	realiz
23	data	trade	econom	economi	deal	compani	develop	govern	bank	tech
24	followan	aku	aja	yang	sama	mau	ini	ada	jfb	bot
25	uyghur	xinjiang	camp	human	muslim	forc	prison	govern	uighur	genocid
26	iran	kill	attack	home	american	countri	dog	border	born	gun
27	space	launch	mission	crew	flight	bbc	photo	test	fly	imag
28	stori	celebr	star	watch	award	movi	share	congratul	connect	congrat
29	food	eat	water	cat	drink	cook	hand	dog	chicken	check
30	museumarch	clevelanda..	cmaopenacc..	clevelanda..	centuri	chineseart	paint	tea	pop	flower
31	research	war	tibetan	independ	tibet	western	histori	west	armi	cia
32	polic	black	protest	white	cop	offic	arrest	matter	murder	kill
33	coronaviru	wuhan	outbreak	spread	infect	death	report	quarantin	covid19	govern
34	free	life	gift	success	easi	guid	practic	essenti	step	mind
35	2020	gizmochina	launch	huawei	pro	phone	xiaomi	camera	appl	featur
36	market	stock	compani	bui	week	price	million	move	month	sell
37	american	immigr	s386	countri	worker	job	indian	passs386	senat	card
38	charact	fuck	feel	read	watch	game	draw	cross	friend	god
39	covid	mask	pandem	test	covid19	health	medic	coronaviru	wear	worker
40	bad	racist	white	wrong	talk	hate	gui	left	stop	blame

beta: 0.09, 0.06, 0.03

Example of tweeted text for topic 28 "Blame CCP"

- "#EvilCCP engineered #CCPVirus #SarsCov2 and released it to wuhan and the world intentionally. Please stand up and hold #CCP accountable. #CCPLiedPeopleDied"
- "#EvilCCP has attacked the American. #CCPLiedAmericansDied"

- "Why is no one talking about punishing the Chinese governemnt for the ChinaVirus They are destablizing the world economy because of their lies deceit and unsanitary conditions They need to pay a very steep price for what they are doing to our country Make China pay"

- "The virus was released by the Chinese Communist Party They killed the Chinese killed people in other countries all over the world [...] Many naive American friends ask me why did the CCP create ht_CCPVirus to hurt its people The devil has no humanity and target US The CCP s evil logic China can die millions people still one billion left How about US CCP wants to rule the world [...] ht_CCP is terrorist So sorry [...] I am Chinese American this is virus from China made by the C4 lab in Wuhan Not only has the CCP brought huge disaster to Chinese people also deceiving world with fake data So sorry Not flu take care everyone Biochemical war ht_CCPVirus ht_CCP is terrorist ht_CCP virus [...]

- "ht_ChinaLiedPeopleDied Yes of course I mean the Chinese government not the Chinese people Can t believe this disclaimer is necessary every freaking time The Chinese Communist Party suppressed initial reports on the Chinese Virus

- "The #CCP banner is scary. You said you have controlled the #CoronaVirus? [...] Has #CCP controlled the #virus in real or just controlled the people? [...] The CCP commits atrocities against the Chinese people and the world. Do not fall victim to their propaganda. The virus is a danger to the Chinese people and all of us because of the CCP. Therefore Wuhan Coronavirus is from the CCP."

D.6 Descriptive Statistics

Table D3: Descriptive Statistics for the White Group

Variable	Observations	Mean	St.Dev.	Median	Min	Max
Panel of users-day:						
Anti-Chinese Slurs	1910550	0.0036	0.0596	0	0	1
Abusive Language	1910550	0.0279	0.1647	0	0	1
Sample daily user average:						
Topic Discrimination	235	0.0164	0.0078	0.0144	0.0109	0.0819

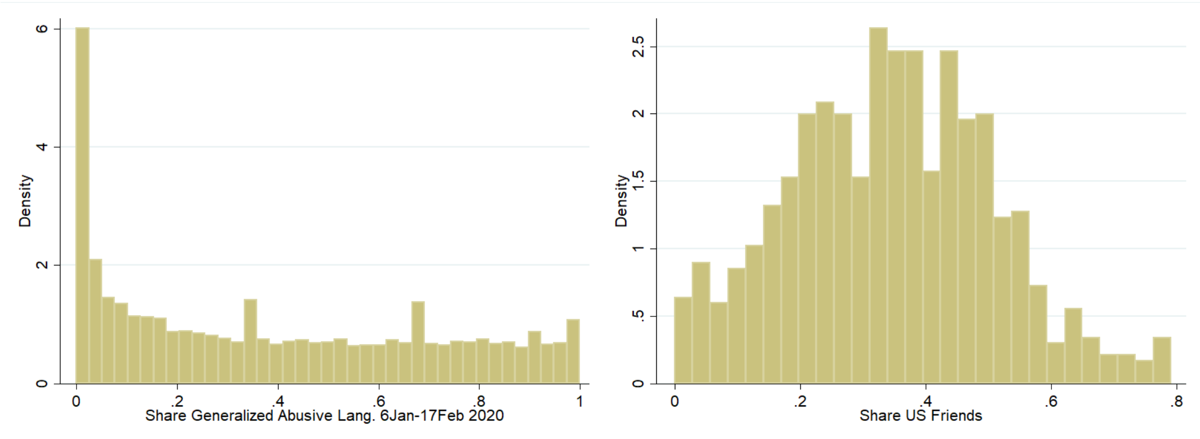
Notes: Observations are users of the White group in each day in the section *Panel of users-day* and daily averages of users of the White group in the section *Sample daily user average*.

Table D4: Descriptive Statistics for the Chinese Group

Variable	Observations	Mean	St.Dev.	Median	Min	Max
Panel of users-day:						
Chinese/Asian American	195520	0.0070	0.0834	0	0	1
Chinese/Asian American + We	195520	0.0014	0.0370	0	0	1
Chinese/Asian American No Report	195520	0.0051	0.0712	0	0	1
Assimilation	195520	0.0349	0.1834	0	0	1
Assimilation No Feel	195520	0.0317	0.1752	0	0	1
Assimilation American	195520	0.0345	0.1824	0	0	1
Sample daily user average:						
T. Blame CCP	235	0.0255	0.0021	0.0254	0.0214	0.0330

Notes: Observations are users of the Chinese group in each day in the section *Panel of users-day* and daily averages of users of the Chinese group in the section *Sample daily user average*.

Figure D5: Heterogeneity of Users of the White and Chinese Groups



(a) White, Share Generalized Abusive Language (b) Chinese, Share of US Friends

Notes: Panel a presents the distribution of the share of generalized abusive language (abusive language not specifically directed against the Chinese community) for users of the White group from January 6 to February 17, 2020. Panel b presents the distribution of the share of US friends for users of the Chinese group. This information was retrieved in the period 20-25 April, 2020.

D.7 Further Results and Robustness

D.7.1 Increasing Discrimination of "White Americans"

Table D5: Discrimination: Trend Break Model Estimates

Dep. Var.	Anti-Chinese Slurs		Abusive Language		Discrimination T.	
	(1)	(2)	(3)	(4)	(5)	(6)
From March 9	0.0020***	0.0001	0.0309***	0.0316**	0.0040***	0.0146
	(0.0007)	(0.0028)	(0.0102)	(0.0127)	(0.0010)	(0.0092)
From March 17	0.0113**	0.0203***	0.0722***	0.1034***	0.0250***	0.0452***
	(0.0047)	(0.0063)	(0.0194)	(0.0192)	(0.0077)	(0.0116)
Date	-0.0000*	0.0003	-0.0008**	-0.0005	0.0000	-0.0000
	(0.0000)	(0.0003)	(0.0003)	(0.0011)	(0.0000)	(0.0000)
From March 9 × [Date-March9]	0.0008***	0.0006	0.0039	0.0032	0.0000	0.0000
	(0.0001)	(0.0004)	(0.0024)	(0.0025)	(0.0000)	(0.0000)
From March 17 × [Date-March17]	-0.0012***	-0.0029***	-0.0074***	-0.0120***	-0.0000***	-0.0000**
	(0.0002)	(0.0007)	(0.0025)	(0.0029)	(0.0000)	(0.0000)
Observations	455280	341460	349590	276420	45	35
Clusters	56	42	43	34		
Adj. R2	0.005	0.008	0.026	0.035	0.359	0.539
Day and Month Dummies		✓		✓		✓

Notes: We consider the sample of tweets of users of the White group. In columns 1 to 4 the unit of observation is at user-day level and the dependent variable is a dummy taking value 1 if the user tweeted using Chinese slurs (columns 1-2) and a dummy taking value 1 if the user tweeted using abusive language and the keyword "Chinese" (columns 3-4). In columns 5-6 the unit of observation is at day level and the dependent variable is the average of the share of text on the topic "Discrimination" against the Chinese computed within the sample of users who tweeted that day. Results are OLS trend break model estimates, odd specifications are unconditional, while even specifications control for dummies for week days and month of the year. Standard errors clustered by date in columns 1-4 and robust in columns 5-6; *** p<0.01, ** p<0.05, * p<0.1.

Table D6: Robustness Discrimination: RDD Estimates Excluding Political and Activist Users

Panel A: Cutoff March 9, 2020 - Excluding Users w/Political+Activist Keywords in Bio			
Dep. Var.	Anti-Chinese Slurs	Abusive Language	Discrimination T.
	(1)	(2)	(3)
RD_Estimate	0.0025	0.0292	0.0027
	(0.0012)	(0.0056)	(0.0001)
Robust P-value	0.2233	0.0005	0.0000
Observations Left	2519	2748	10
Observations Right	1832	1832	4
Polynomial Order	1	1	1
Band. Method	msetwo	msetwo	msetwo
Band. Left	11.094	12.324	10.431
Band. Right	7.000	7.000	3.109
Day and Month Dummies	✓	✓	✓

Panel B: Cutoff March 17, 2020 - Excluding Users w/Political+Activist Keywords in Bio			
Dep. Var.	Anti-Chinese Slurs	Abusive Language	Discrimination T.
	(1)	(2)	(3)
RD_Estimate	0.0154	0.0678	0.0294
	(0.0031)	(0.0092)	(0.0038)
Robust P-value	0.0009	0.0000	0.0000
Observations Left	1603	1603	7
Observations Right	3664	4580	20
Polynomial Order	1	1	1
Band. Method	msetwo	msetwo	msetwo
Band. Left	7.000	7.000	7.000
Band. Right	15.733	19.930	19.950
Day and Month Dummies	✓	✓	✓

Notes: We consider the subsample of tweets of "White American" users whose Twitter bio does not include keywords related to politics and activism, before March 17 in panel A and after March 9 in Panel B. In both Panels the unit of observation in columns 1-2 is at user-day level and the dependent variable is a dummy taking value 1 if the user tweeted using Chinese slurs in columns 1 and a dummy taking value 1 if the user tweeted using abusive language and the keyword "Chinese" in column 2. In both panels in column 3 the unit of observation is at day level and the dependent variable is the average of the share of text on the topic "Discrimination" against the Chinese computed within the sample of users who tweeted that day. Results are local polynomial estimates using March 9 as cutoff in Panel A and March 17 as cutoff in Panel B, all specifications control for dummies for week days and month of the year. Except for specification 3 that uses robust standard errors, standard errors reported in parentheses are clustered by date. Statistical significance is computed based on the robust P value. Different bandwidths on each side of the cutoff are derived under the MSE procedure using a linear polynomial and a uniform kernel.

Table D7: Robustness Discrimination: RDD Estimates Only Political and Activist Users

Panel A: Cutoff March 9, 2020 - Only Users w/Political+Activist Keywords in Bio			
Dep. Var.	Anti-Chinese Slurs	Abusive Language	Discrimination T.
	(1)	(2)	(3)
RD_Estimate	0.0015	0.0460	0.0041
	(0.0005)	(0.0093)	(0.0012)
Robust P-value	0.0107	0.0060	0.5425
Observations Left	86911	63208	6
Observations Right	63208	63208	8
Polynomial Order	1	1	1
Band. Method	msetwo	msetwo	msetwo
Band. Left	11.825	8.110	6.707
Band. Right	7.000	7.000	7.000
Day and Month Dummies	✓	✓	✓

Panel B: Cutoff March 17, 2020 - Only Users w/Political+Activist Keywords in Bio			
Dep. Var.	Anti-Chinese Slurs	Abusive Language	Discrimination T.
	(1)	(2)	(3)
RD_Estimate	0.0192	0.1183	0.0509
	(0.0053)	(0.0139)	(0.0067)
Robust P-value	0.0004	0.0000	0.0000
Observations Left	55307	55307	7
Observations Right	181723	142218	21
Polynomial Order	1	1	1
Band. Method	msetwo	msetwo	msetwo
Band. Left	7.000	7.000	7.000
Band. Right	22.766	17.855	20.061
Day and Month Dummies	✓	✓	✓

Notes: We consider the subsample of tweets of "White American" users whose Twitter bio includes keywords related to politics and activism, before March 17 in panel A and after March 9 in Panel B. In both Panels the unit of observation in columns 1-2 is at user-day level and the dependent variable is a dummy taking value 1 if the user tweeted using Chinese slurs in columns 1 and a dummy taking value 1 if the user tweeted using abusive language and the keyword "Chinese" in column 2. In both panels in column 3 the unit of observation is at day level and the dependent variable is the average of the share of text on the topic "Discrimination" against the Chinese computed within the sample of users who tweeted that day. Results are local polynomial estimates using March 9 as cutoff in Panel A and March 17 as cutoff in Panel B, all specifications control for dummies for week days and month of the year. Except for specification 3 that uses robust standard errors, standard errors reported in parentheses are clustered by date. Statistical significance is computed based on the robust P value. Different bandwidths on each side of the cutoff are derived under the MSE procedure using a linear polynomial and a uniform kernel.

D.7.2 Reaction of the Chinese Minority

Table D8: Assimilation: Trend Break Model Estimates

Dep. Var.	Chinese/Asian American		Assimilation		Blame CCP Topic	
	(1)	(2)	(3)	(4)	(5)	(6)
From March 9	0.0007	-0.0027	0.0030	0.0048	0.0009	0.0018
	(0.0029)	(0.0034)	(0.0040)	(0.0042)	(0.0010)	(0.0012)
From March 17	0.0127***	0.0144***	0.0139***	0.0157***	0.0036***	0.0041***
	(0.0032)	(0.0037)	(0.0042)	(0.0048)	(0.0012)	(0.0014)
Date	0.0001	0.0005*	0.0003	-0.0006	0.0002***	0.0000
	(0.0001)	(0.0003)	(0.0004)	(0.0007)	(0.0001)	(0.0002)
From March 9 × [Date-March9]	0.0000	-0.0001	0.0009	0.0019**	-0.0002	-0.0001
	(0.0006)	(0.0007)	(0.0008)	(0.0009)	(0.0002)	(0.0002)
From March 17 × [Date-March17]	-0.0005	-0.0013*	-0.0025***	-0.0033***	-0.0000	-0.0000
	(0.0006)	(0.0007)	(0.0007)	(0.0006)	(0.0002)	(0.0002)
Observations	37440	33280	34112	29120	49	44
Clusters	45	40	41	35		
Adj. R2	0.002	0.003	0.002	0.001	0.530	0.566
Day and Month Dummies		✓		✓		✓

Notes: We consider the sample of tweets of users of the Chinese group. In columns 1 to 4 the unit of observation is at user-day level and the dependent variable is a dummy taking value 1 if the user tweeted using the keywords "Chinese American" or "Asian American" (columns 1-2) and a dummy taking value 1 if the user tweeted assimilation content (columns 3-4). In columns 5-6 the unit of observation is at day level and the dependent variable is the average of the share of text on the topic "Blame CCP" computed within the sample of users who tweeted that day. Results are OLS trend break model estimates, odd specifications are unconditional, while even specifications control for dummies for week days and month of the year. Standard errors clustered by date in columns 1-4 and robust in columns 5-6; *** p<0.01, ** p<0.05, * p<0.1.